

- 1) You are a beef farmer deciding on your ration for steers on your feedlot. You used your records and data from a local consultant to estimate the amount of hay and grain to put 500 pounds of gain on your cattle. You have constructed the table below. Fill in the table below and answer the following questions.

Table for part a when treating hay as input X and grain as input Y

Hay (X) (lbs/animal)	Grain (Y) (lbs/animal)	Marginal Rate of Technical Substitution	Price Ratio
2,700	880		<i>Part a Part b</i>
2,380	1,050	0.531	1.111 1.25
2,170	1,170	0.571	1.111 1.25
1,960	1,320	0.714	1.111 1.25
1,840	1,470	1.250	1.111 1.25
1,680	1,680	1.313	1.111 1.25
1,570	1,860	1.636	1.111 1.25

Table for part a when treating hay as input Y and grain as input X

Hay (Y) (lbs/animal)	Grain (X) (lbs/animal)	Marginal Rate of Technical Substitution	Price Ratio
2,700	880		<i>Part a Part b</i>
2,380	1,050	1.882	0.90 0.80
2,170	1,170	1.750	0.90 0.80
1,960	1,320	1.400	0.90 0.80
1,840	1,470	0.800	0.90 0.80
1,680	1,680	0.762	0.90 0.80
1,570	1,860	0.611	0.90 0.80

You can do either version of table.

- a) If the price of grain is \$180.00/ton and the price of hay is \$200.00/ton, what is the economically optimal amount of grain and hay to feed to your cattle? (You may need to interpolate between entries on the table.)

For the 1st table (hay as X and grain as Y)

First MRTS entry as an example: $-\Delta Y/\Delta X = -(1050 - 880) / (2380 - 2700) = 0.531$

Price ratio is $r_x / r_y = 200 / 180 = 1.111$

For the 2nd table (grain as X and hay as Y)

First MRTS entry as an example: $-\Delta Y/\Delta X = -(2380 - 2700) / (1050 - 880) = 1.882$

Price ratio is $r_x / r_y = 180 / 200 = 0.900$

Optimal input use is where MRTS = price ratio. The price ratio is 1.111 or 0.900 between entries on the table noted by the **maroon** arrow and box, so use linear interpolation. The formula is $B_{btwn} = B_0 + (A_{btwn} - A_0) \frac{B_1 - B_0}{A_1 - A_0}$, where A is the MRTS and B is Hay or Grain.

Using the top table: $Hay_{btwn} = 1960 + (1.111 - 0.714) \frac{1840 - 1960}{1.25 - 0.714} = 1,871 \text{ lbs}$

$Grain_{btwn} = 1320 + (1.111 - 0.714) \frac{1470 - 1320}{1.25 - 0.714} = 1,431 \text{ lbs}$

Using the bottom table: $Hay_{btwn} = 1960 + (0.900 - 1.400) \frac{1840 - 1960}{0.8 - 1.4} = 1,860 \text{ lbs}$

$Grain_{btwn} = 1320 + (0.875 - 0.900) \frac{1470 - 1320}{0.8 - 1.4} = 1,445 \text{ lbs}$

Note that you can use either version of the table and do not have to do both. Also notice how the two tables each give you different answers. This is due to the inexact nature of the linear interpolation, because the actual underlying curves are non-linear curves. This is another reason why using calculus is a preferred method.

- b) How does your answer change if the price of grain increases to \$200.00/ton and the price of hay increases to \$250/ton? (You may need to interpolate between entries on the table.)

For the 1st table: Price ratio is $r_x / r_y = 250 / 200 = 1.25$

For the 2nd table: Price ratio is $r_x / r_y = 200 / 250 = 0.80$

Optimal input use is where $MRTS = \text{price ratio}$. This price ratio is exactly on an entry in the table as noted by the **light green** arrow, so use Hay of 1,840 and Grain of 1,470.

- 2) You have a summer internship with a swine company. Their scientists give you the following production function for the final weight for feeder pigs bought to feed and sell as hogs:

$$W = -1750 + 7C + 8S - 0.08C^2 - 0.06S^2 + 0.11CS,$$

where W is each hog's final weight in pounds, C is pounds of ground corn consumed and S pounds of soybean meal consumed. Answer the following questions:

- a) If the price of hogs is \$90/cwt (hundredweight), the price of ground corn is \$180/ton, and the price of soybean meal is \$360/ton, what are each of these prices in \$ per pound? Note 1 cwt equals 100 pounds and a ton is 2,000 pounds.

$$\frac{\$90}{1 \text{ cwt}} \times \frac{1 \text{ cwt}}{100 \text{ lbs}} = \$0.90/\text{lb}, \quad \frac{\$180}{1 \text{ ton}} \times \frac{1 \text{ ton}}{2,000 \text{ lbs}} = \$0.09/\text{lb}, \quad \frac{\$360}{1 \text{ ton}} \times \frac{1 \text{ ton}}{2,000 \text{ lbs}} = \$0.18/\text{lb}$$

- b) How much ground corn and soybean meal is it economically optimal to feed the feeder pigs? First write out the profit equation using the prices in \$ per pound. Second, take the two first derivatives and set them equal to zero. Third, solve these two equations for S and C. Fourth, **check the second order conditions**.

$$\pi = 0.9(-1750 + 7C + 8S - 0.08C^2 - 0.06S^2 + 0.11CS) - 0.09C - 0.18S$$

The partial derivatives of this function with respect to C and S give the following FOC's:

$$\text{FOC for C: } 0.9(7 - 0.16C + 0.11S) - 0.09 = 0$$

$$\text{FOC for S: } 0.9(8 - 0.12S + 0.11C) - 0.18 = 0$$

Several ways exist to solve these two equations for the two variables C and S. I lay out a process below, but it is not the only way. Rearranging the FOC for C gives

$$0.9(7 - 0.16C + 0.11S) = 0.09$$

$$7 - 0.16C + 0.11S = 0.09/0.9 = 0.10$$

$$6.9 + 0.11S = 0.16C$$

$$C = 6.9/0.16 + (0.11/0.16)S \quad \text{or} \quad C = 43.1 + 0.688S$$

Substitute this into the FOC for S and solve for S:

$$0.9(8 - 0.12S + 0.11C) - 0.18 = 0$$

$$8 - 0.12S + 0.11C = 0.18/0.9 = 0.2$$

$$8 - 0.12S + 0.11(43.1 + 0.688S) = 0.2$$

$$8 - 0.12S + 4.74 + 0.076S = 0.2$$

$$12.74 - 0.044S = 0.2$$

$$12.74 - 0.2 = 12.54 = 0.044S$$

$S = 12.54/0.044 = 285$ lbs of soybean meal, but answers will vary slightly due to rounding differences. Without rounding, the answer is 282.676... lbs

Use this S to find C :

$C = 43.1 + 0.688S = 43.1 + 0.688(285) = 239.2$ lbs of corn, but again answers will vary slightly due to differences in rounding. Without rounding the answer is 237.4648... lbs

First I report the second derivatives of π , then check the SOC's.

$$d^2\pi/dC^2 = 0.9(-0.16) = -0.144$$

$$d^2\pi/dS^2 = 0.9(-0.12) = -0.108$$

$$d^2\pi/dCdS = 0.9(0.11) = +0.099$$

SOC's

$$d^2\pi/dC^2 = -0.144 < 0 \text{ satisfied}$$

$$d^2\pi/dS^2 = -0.108 < 0 \text{ satisfied}$$

$$(d^2\pi/dC^2)(d^2\pi/dS^2) - (d^2\pi/dCdS)^2 = (-0.144)(-0.108) - (0.099)^2 = 0.00575 > 0$$

All three SOC's are satisfied for a maximum.

- c) When feeding these levels of S and C , what is the final weight of the hogs?

Substitute the optimal C and S found in part a into the weight function:

$$W = -1750 + 7C + 8S - 0.08C^2 - 0.06S^2 + 0.11CS$$

$$W = -1750 + 7(239.2) + 8(285) - 0.08(239.2)^2 - 0.06(285)^2 + 0.11(239.2)(285)$$

$W = 252.5$ lbs is the final weight. Answers will vary slightly due to rounding differences.

Without rounding, the answer is 252.0 lbs.

- d) When feeding these levels, what is the net return per feeder pig? Assume net return per pig is $\pi = pW - r_cC - r_sS - 85$, where p , r_c and r_s are the prices of hogs, ground corn and soybean meal in \$ per pound and \$85 is all other costs as \$ per pig.

Substitute the results from above into the given returns function:

$$\pi = pW - r_cC - r_sS - 85$$

$\pi = 0.9(252.5) - 0.09(239.2) - 0.18(285) - 85 = \69.42 per pig. Answers will vary slightly due to rounding differences. Without rounding, the answer is \$69.52/pig