AAE 320 Problem Set #2 **Name:_____KEY_____** **Due Sep 23, 2020**

1) You are a crop consultant helping a grain farmer in Wisconsin's Central Sands determine his economically optimal irrigation level. The last few years you ran on-farm irrigation experiments to create the first two columns in the table below for his average yield for different levels of total water (rainfall plus irrigation) during the growing season.

Fill in the table below and answer the following questions.

Total Water (acre inches)	Average Corn Yield (bu/ac)	Marginal Product (bu/ac)	Value of the Marginal Product ($/ac)
12	265.0	---	---
14	273.0	*4.00*	*12.00*
16	280.6	*3.80*	*11.40*
18	287.6	*3.50*	*10.50*
20	293.6	*3.00*	*9.00*
22	298.4	*2.40*	*7.20*
24	302.0	*1.80*	*5.40*
26	304.6	*1.30*	*3.90*

<u>Marginal Product (MP)</u> is ΔQ/ΔX, or change in output divided by change in the input level. For the first two rows, ΔQ/ΔX = (273.0 – 265.0)/(14 – 12) = 4.00 bu/acre. The rest are in the table.

<u>Value of the Marginal Product (VMP)</u> is the price of output (here corn) times the Marginal Product. For the second row of the table, this is $3.00 per bu times the MP of 4.0 bu per acre, or 3.00 x 4.0 = $12.00. The rest are in the table.

When he plants, he forward contracts to sell his corn at a price of $3.00/bu. You estimate that his cost of pumping water is $9.00 per acre inch.

a) Suppose your client typically makes sure his corn gets 22 inches of water and this year he wants to go with 24 inches. Answer the following questions in order to explain to him why this is not a good plan if he wants to maximize his net returns:

i. When he goes from 22 to 24 inches, how much does his average yield increase?

Yield increase is Q with 24 inches minus Q with 22 inches, or 302.0 – 298.4= 3.6 bu.

ii. How much is this extra yield worth?

3.6 bu x $3/bu = $10.80

iii. How much will it cost him to pump these extra 2 inches of irrigation water?

Cost is $9.00 per inch, so 2 inches x $9.00 = $18.00.

iv. Is adding these two inches of water a good idea if he wants to maximize his net returns? Why or why not?

No, he would pay $18.00 per acre to pump these two more inches, but only gain $10.80 more in corn. It does not make sense to spend $18.00 to get $10.80 if your goal is to make money.

b) Assume he wants to maximize his net returns.
i. How much water should he plan make sure his corn has (you may need to interpolate)?

In class, we showed that the optimal input use occurs where VMP = r, so here, where VMP = $9.00 per acre, which occurs in the Table at <u>*20 inches of water per acre*</u>*.*

ii. At this amount of water, what is his average corn yield and what is its value at $3.00/bu?

With 20 inches of water, yield = 293.6 bu/A, then at $3.00/bu = $880.80/A in revenue

iii. If it usually rains 12 inches during the growing season, how much water will he have to pump? What will it cost to pump this much water if pumping costs $9.00 per acre inch?

The goal of 20 inches – 12 inches in rain = 8 inches to pump, so at $9.00 per inch x 8 inches/A = $72.00 per acre

iv. Assuming he has $600/ac in other costs, what are his net returns per acre?

Net returns = *$880.80/A – $72.00/A – $600.00/A = $208.80/A*

c) Suppose he replaces his current high-pressure irrigation system with a more efficient low-pressure system that uses less energy and wastes less water, and as a result, pumping costs decrease to $5.40 per acre inch. Assume he still wants to maximize his net returns.

How much water should he plan on making sure his corn has?

If the cost of the input water is $5.40/acre inch of water with the new system, then the optimality condition VMP = r occurs at 24 inches of water.

d) Suppose a group wants to reduce agricultural use of water in the area and to do so, they are offering farmers subsidies to replace older high-pressure irrigation systems with new more efficient low-pressure systems that use less energy and waste less water and so have lower pumping costs per acre-inch. (In other words, the older systems are like in parts 1a and 1b and the newer systems are like in part 1c.) Their logic for these subsidies is that with these new more energy efficient systems, farmers will pump less water per acre. These new systems are more expensive to buy and install than a conventional system, but once in place, the new system is more energy efficient and so has a <u>lower cost</u> to pump one acre inch of water. Comment on how effective you think this program will be at reducing water use for crop production – <u>How does economically optimal input use respond to a lower input price</u>?

The new more efficient system actually reduces the marginal cost of pumping water, which is the same as decreasing the "price" of the water input, and so farmers will use more of the input, just like the difference in the answers in problem 1b versus 1c. When water decreases in cost from $9.00 per inch to $5.40 per inch (a 40% decline in cost), optimal water use went from 20 inches per acre to 24 inches per acre (a 20% increase). This type of response happens commonly today in energy demand and computer speed demand – make a more efficient system and people actually use more energy and demand more computer speed. It's been known since the 1800s and is called Jevon's Paradox in the history literature or the Rebound Effect in the modern energy literature (Google it if you are interested).

2) You are the same crop consultant, but now determining the economically optimal nitrogen rate for your client's corn fields using Extension nitrogen fertilizer recommendations. You will first derive the optimal rate using calculus, and then using an online tool.

Suppose expected corn yield is $Y = 165 + 0.996N - 0.0023N^2$, where Y is corn yield in bushels per acre and N is the total nitrogen applied as pounds of N per acre. Assume the expected corn price is \$3.20/bu and Urea N nitrogen fertilizer solution (45% N by weight) costs \$400/ton.

a) The production function uses nitrogen in pounds of N per acre, but the price is \$400 per ton of fertilizer. Convert the price of the fertilizer from \$400/ton of fertilizer to \$/lb of N. If urea N nitrogen fertilizer solution (45% N by weight) costs \$400/ton, what is the price of nitrogen N in \$ per pound?

Based on the numbers above, the calculations are:

$$\frac{\$400}{1 \text{ ton of Fert.}} x \frac{1 \text{ ton of Fert.}}{2{,}000 \text{ pounds of Fert.}} x \frac{1 \text{ pound of Fert.}}{0.45 \text{ pounds of N}} = \frac{\$0.44}{1 \text{ pound of N}}$$

or \$0.44/pound for N.

b) Using the price for N, a corn price of \$3.20/bu, and the given production function, write the profit (π) equation for the farmer's net return per acre.

$\pi(X) = pQ - rX$, where $Q = f(X)$. Here X is nitrogen, or N, so we write
$\pi(N) = 3.20(165 + 0.996N - 0.0023N^2) - 0.44N$

c) Using calculus, what N rate maximizes profit for the farmer? Give the first order condition, solve it for the optimal N rate, and then check the second order condition.

$\pi(N) = 3.20(165 + 0.996N - 0.0023N^2) - 0.44N$

The first order condition (FOC): $d\pi/dN = 3.20(0.996 - 0.0046N) - 0.44 = 0$.

Solving FOC for N gives N = 186.6 or 187 lbs of N per acre.

The second order condition (SOC) is $d^2\pi/dN^2 = -0.01472 < 0$, which satisfies the condition for a maximum.

d) At this nitrogen rate, what is his yield (bu/ac)? What is his per acre revenue? How much will it cost (\$/ac) to buy this much nitrogen fertilizer? If he has \$700/ac in other costs, what are his per acre net returns (profit) at this nitrogen rate?

Substitute N = 187 into the production function $Y = 165 + 0.996N - 0.0023N^2$
$Y = 165 + 0.996(187) - 0.0023(187)^2 = 270.8$ bu per acre

Revenue = Corn Price x Yield = 3.20 x 270.8 = \$866.56 per acre

Fertilizer Cost = N Price x N = 0.44 x 187 = \$82.28 per acre

Profit = Revenue – Cost = \$866.56 – \$82.28 – \$700.00 = \$84.28 per acre

e) Assume you find some cheaper urea fertilizer for a price of \$360/ton, and the price of corn is now \$3.30/bu. Using the same production function, what is the optimal nitrogen rate to apply? What is the expected yield at this nitrogen rate? Again, assuming \$700/ac in other costs, what are his per acre returns at this nitrogen rate?

Convert \$360/ton of fertilizer to \$/lb of N:

$$\frac{\$360}{1 \text{ ton of Fert.}} x \frac{1 \text{ ton of Fert.}}{2{,}000 \text{ pounds of Fert.}} x \frac{1 \text{ pound of Fert.}}{0.45 \text{ pounds of N}} = \frac{\$0.40}{1 \text{ pound of N}}$$

$\pi = 3.30(165 + 0.996N - 0.0023N^2) - 0.40N - 700$

The first order condition (FOC): $d\pi/dN = 3.30(0.996 - 0.0046N) - 0.40 = 0$.

Solving FOC for N gives N = 190.2 or 190 lbs of N per acre.

The second order condition (SOC) is $d^2\pi/dN^2 = -0.01518 < 0$, which satisfies the condition for a maximum.

Substitute N = 190 into the production function $Y = 165 + 0.996N - 0.0023N^2$
$Y = 165 + 0.995(190) - 0.0023(190)^2 = 271.2$ bu per acre

Revenue = Corn Price x Yield = 3.30 x 271.2 = $894.96 per acre

Fertilizer Cost = N Price x N = 0.40 x 190 = $76.00 per acre

Profit = Revenue – Cost = $894.96 – $76.00 – $700.00 = $118.96 per acre

f) Many Midwestern states use this method to develop Extension recommendations for nitrogen fertilizer use in corn. Go to http://cnrc.agron.iastate.edu/ for the Corn Nitrogen Rate Calculator and read the "About" tab, especially "Definitions." Next, choose "Single Price" and then choose "Wisconsin" for the state, "Corn following Soybean" for the rotation, and "Irr. Sands" for the soil type (irrigated sands). Choose the right form of nitrogen "Urea (45% N)" and enter the nitrogen and corn prices from part a and hit the "Calculate" button.

i) Does the price of N in $/lb match your answer in part a? You should answer "Yes" here, if not, go back and get the price right in part a or enter the correct inputs here. Note that the program rounds to the nearest full cent per pound.

You should get $0.44/pound. See Figure 1 below.

ii) At a urea N fertilizer price of $400/ton and a corn price of $3.20/bu, what does the tool indicate is the optimal N rate?

MRTN Rate = 187 pounds of N per acre. See Figure 1 below.

iii) If the Urea N nitrogen fertilizer price is $360/ton and the corn price is $3.30/bu, what does the tool indicate is the optimal N rate?

MRTN Rate = 191 pounds of N per acre. See Figure 1 below.

iv) How do these compare to your answers in parts a and e? They should be very close to the same amount; if not, go back and get things right.

They are basically the same. The tool and equation are within one pound per acre of each other.

The UW Extension Nutrient and Pest Management Program (http://ipcm.wisc.edu/) has developed free apps for iPhones/iPads and Android Smartphones/Tablets, the "Corn N Rate Calculator" and the "N Price Calculator". Search in the iTunes Store or Google Play (or see http://ipcm.wisc.edu/apps/) and download them if you want to. When you do, you will see lots of other interesting apps available as well.

Figure 1. Corn Nitrogen Rate Calculator output for $400/ton urea and $3.20/bu corn

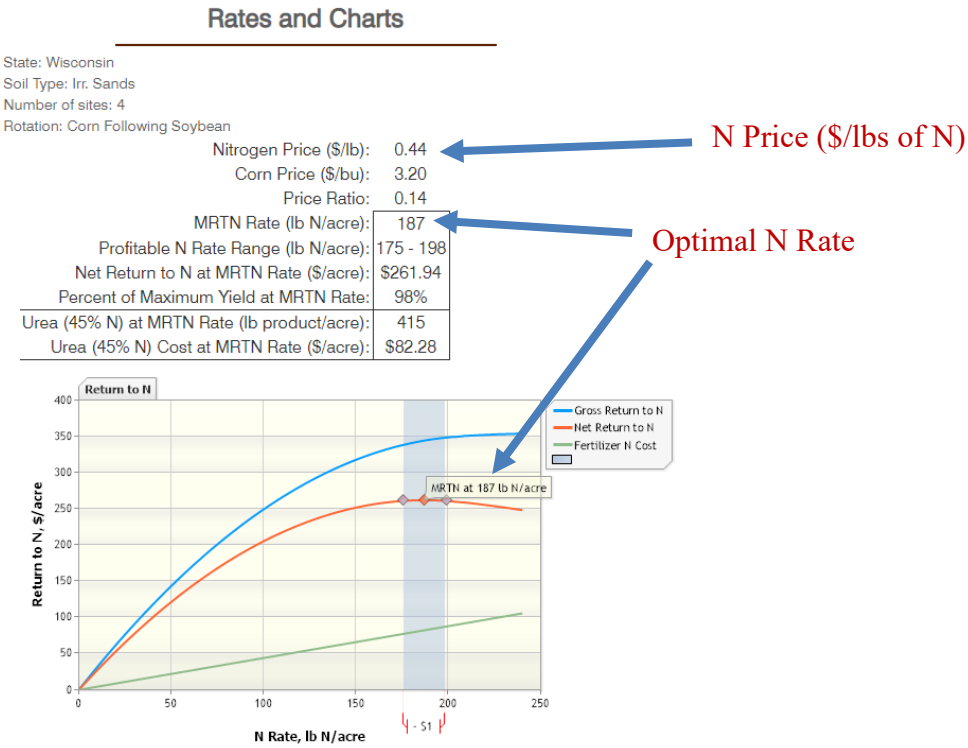

Figure 2. Corn Nitrogen Rate Calculator output for $360/ton urea and $3.30/bu corn