Single Input Production Economics for Farm Management

AAE 320: Farming Systems Management

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Production Economics Learning Goals

- Single and Multiple Input Production Functions
  - What are they and how to use them in production economics and farm management
- Economics to identify optimal input and output combinations
  - How much nitrogen fertilizer do I use for my corn?
  - How much corn will I get if I use this much nitrogen?

- Application of basic production economics to farm management

Production

- Definition: Using inputs to create goods and services having value to consumers or other producers
- Production is what farms do!
- Using land, labor, time, machinery, animals, seeds, fertilizer, water, etc. to grow crops, livestock, milk, eggs, etc.
- Can further process outputs: cheese, jams/jellies, baked goods
- Can produce services: bed and breakfast, orchard/pumpkin farm with hayrides selling the "fall country experience"

Production Function

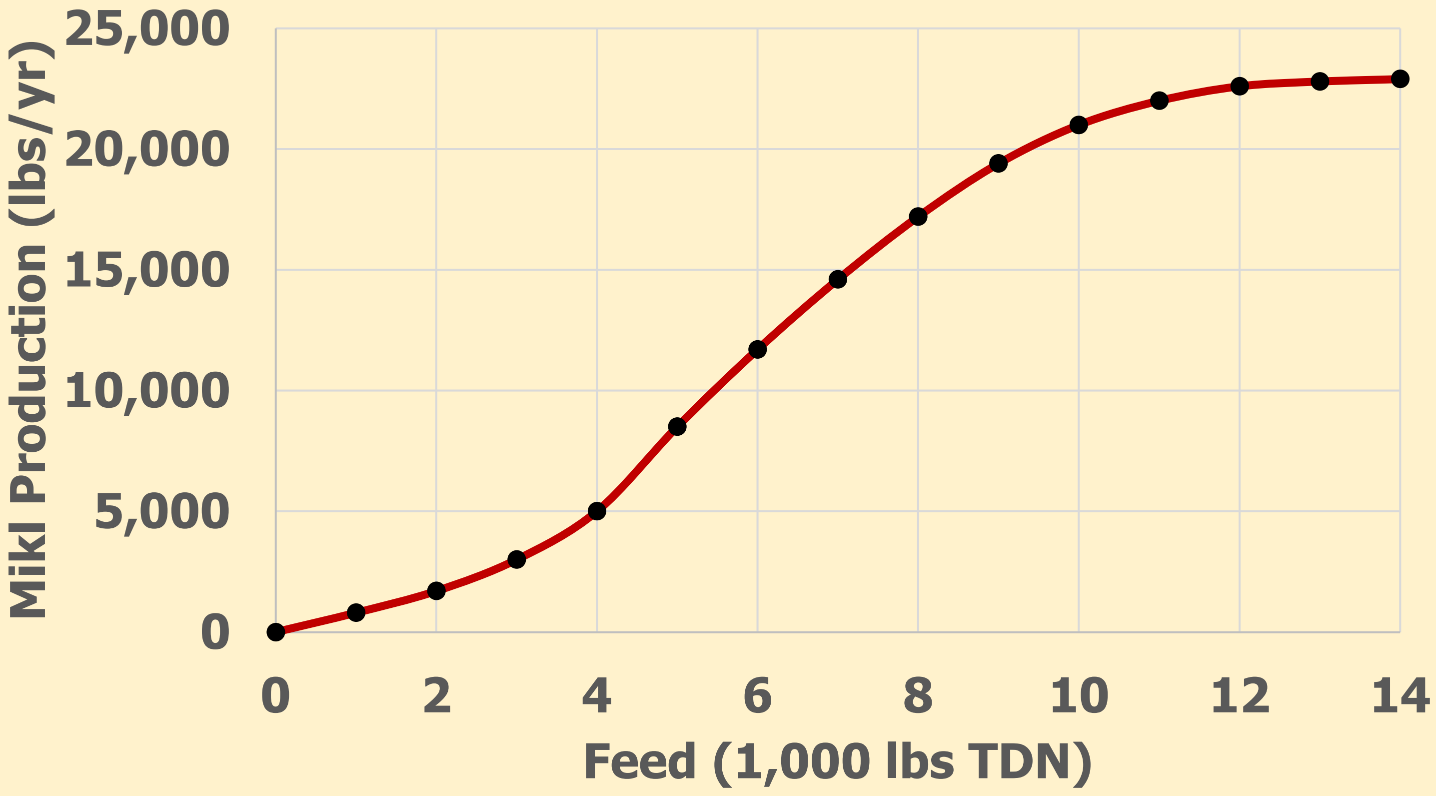
- Production Function: gives the maximum amount of output that can be produced for the given input(s)
- Generally two types:
  - Tabular Form (Production Schedule)
  - Mathematical Function

Feed (1000 lbs TDN/yr)	Milk (lbs/yr)
0	0
1	800
2	1,700
3	3,000
4	5,000
5	8,500
6	11,700
7	14,600
8	17,200
9	19,400
10	21,000
11	22,000
12	22,600
13	22,800
14	22,900

Tabular Form

A table listing the maximum output for each given input level

TDN = total digestible nutrition (feed)



Production Function

- Mathematically express the relationship between input(s) and output
- Single Input, Single Output
  - Milk = f(TDN)
  - $Milk = 50 + 3TDN – 0.2TDN^2$
- Multiple Input, Single Output
  - Milk = f(Corn, Soy)
  - $Milk = 50 + 3Corn – 0.2Corn^2 + 2Soy – 0.1Soy^2 + 0.4CornSoy$

Examples

- Polynomial: Linear, Quadratic, Cubic
  - $Milk = b_0 + b_1TDN + b_2TDN^2$
  - $Milk = -2261 + 2.535TDN - 0.000062TDN^2$

- Many functions are used, depending on the process: Cobb-Douglas, von Liebig (plateau), Exponential, Hyperbolic, etc.

Why Production Functions?

- More convenient & easier to use than tables
- Estimate via regression with the tables of data from experiments
- Increased understanding of production process: identify important factors and how important factor each is
- Allows use of calculus for optimization
- Common activity of agricultural research scientists

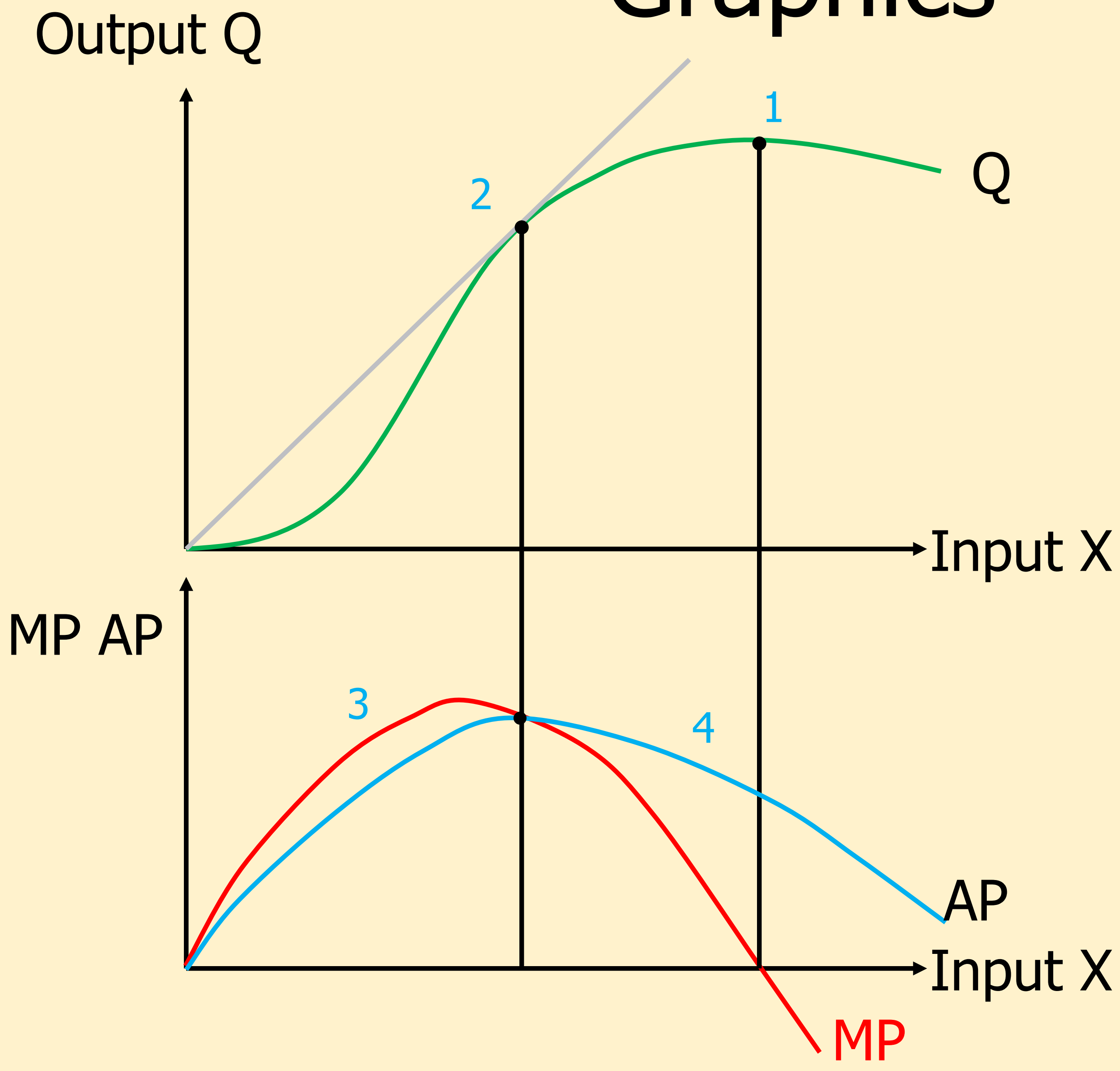
Definitions

- Input: X   Output: Q
- Total Product = Output Q
- Average Product (AP) = Q/X: average output for each unit of the input used
  - Example: you harvest 200 bu/ac corn and applied 100 lbs of nitrogen
  - AP = 200/100 = 2, means on average, you got 2 bu of corn per pound of nitrogen applied
- Graphics: slope of line between origin and the total product curve

Definitions

- Marginal Product (MP) = $\Delta Q/\Delta X$ or derivative dQ/dX:
  - Output Q generated by the last unit of input used or applied
  - Example: corn yield increases from 199 to 200 bu/ac when you increase nitrogen applied from 99 lbs to 100 lbs
  - MP = 1/1 = 1, meaning you got 1 bu of corn from last 1 pound of nitrogen applied
- MP: Slope of total product curve

Graphics



1) MP = 0 when Q at maximum, i.e. slope = 0
2) AP = MP when AP at maximum, at Q where line btwn origin and Q curve tangent
3) MP > AP when AP increasing
4) AP > MP when AP decreasing

MP and AP: Tabular Form

Input	TP	MP	AP
0	0		
1	6	6	6.0
2	16	10	8.0
3	29	13	9.7
4	44	15	11.0
5	55	11	11.0
6	60	5	10.0
7	62	2	8.9
8	62	0	7.8
9	61	-1	6.8
10	59	-2	5.9

$MP = \Delta Q/\Delta X = (Q_2 - Q_1)/(X_2 - X_1)$

$AP = Q/X$

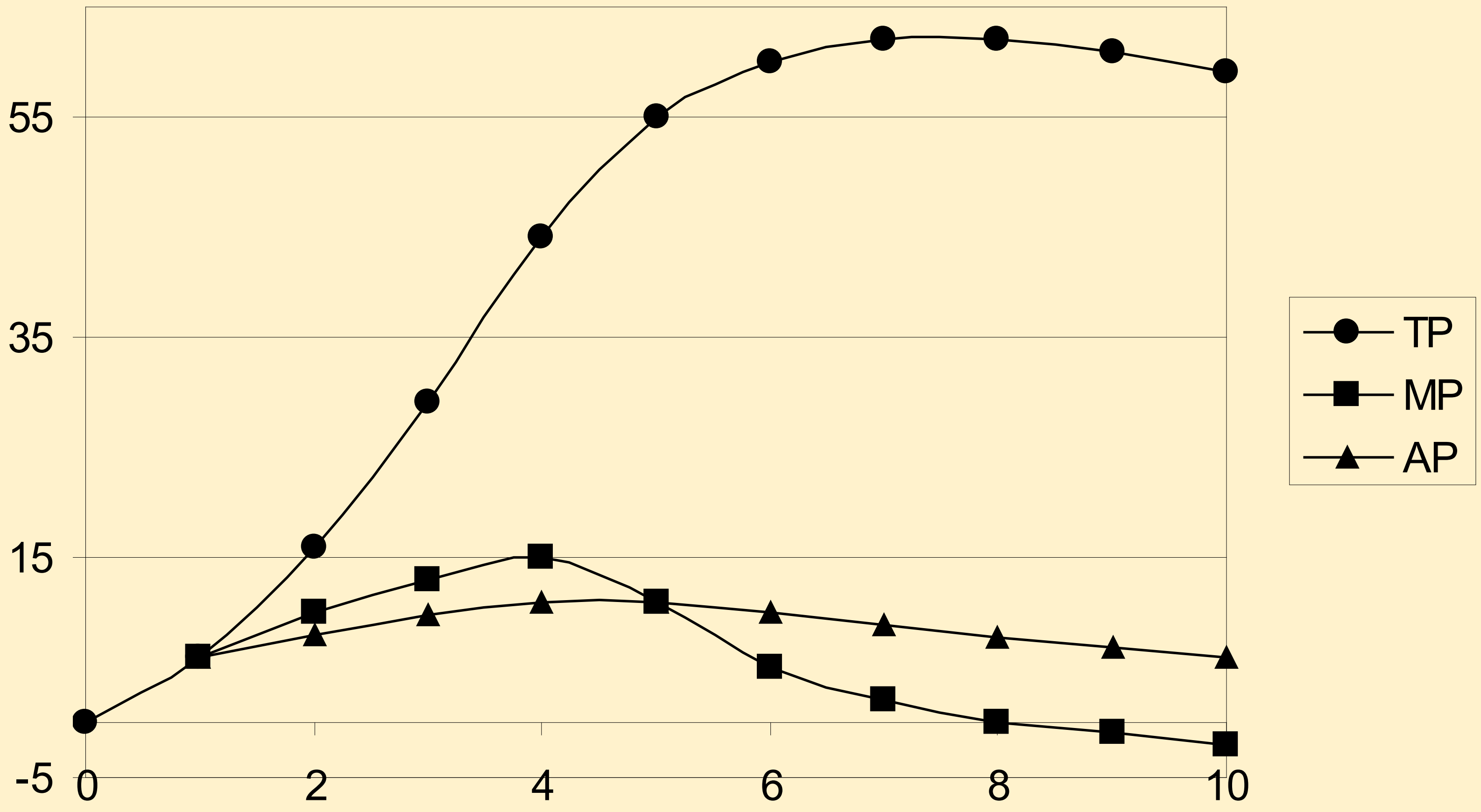
MP: $6 = (6 - 0)/(1 - 0)$

AP: $8.0 = 16/2$

MP: $5 = (60 - 55)/(6 - 5)$

AP: $8.9 = 62/7$

Same Data: Graphically



Think Break #2

Nitrogen (N) and Corn Yield

- Fill in the missing numbers in the table for the Average Product (AP) and the Marginal Product (MP)
- Remember the Formulas

$$MP = \Delta Q/\Delta X$$

$$= (Q_2 - Q_1)/(X_2 - X_1)$$

$$AP = Q/X$$

N	Yield	AP	MP
0	30	---	---
25	45	1.8	0.6
50	75		1.2
75	105	1.4	
100	135	1.35	1.2
125	150		0.6
150	165	1.1	
175	168	0.96	0.12
200	170	0.85	0.08
225	171	0.76	0.04

Law of Diminishing Marginal Product

- Diminishing MP: <u>Holding all other inputs fixed</u>, as you use more and more of one input, eventually the MP starts decreasing
  - The returns to increasing that input eventually start to decrease
- Common in biological, physical and social systems: eventually the marginal product (MP) starts to decrease
  - As you make more and more feed available to a cow, the <u>extra</u> milk produced eventually starts to decrease
  - As add more corn acres (holding all other inputs fixed), the <u>extra</u> corn produced eventually decreases due to less time to plant and for crop care, more time travel between fields, only lower quality land available, …

Transition

- We spent time explaining production functions Q = f(X) and their slope = MP, and AP = Q/X
- Now we can ask: How do we use them?
- How do I decide how much input to use?
  - How much nitrogen should I use for my corn?
  - How many soybean seeds should I plant per acre?
- Choose each input to maximize farmer profit
- We will set it up as an economic problem
- First as partial budget and then use calculus

<table>
<tr><td colspan="4">Suppose you apply 99 pounds of Nitrogen per acre to corn. Should you apply 100 pounds?</td></tr>
<tr><td colspan="2">Benefits</td><td colspan="2">Costs</td></tr>
<tr><td colspan="2"><u>Additional Revenues</u>
Extra yield = 180 bu – 179 bu = 1 bu/acre x $3.00/bu = $3.00/acre (Value of the MP)</td><td colspan="2"><u>Additional Costs</u>
$0.50/lb x 1 lb of N/acre = $0.50/acre (Input price)</td></tr>
<tr><td colspan="2"><u>Costs Reduced</u>
None</td><td colspan="2"><u>Revenues Reduced</u>
None</td></tr>
<tr><td>Total Benefits</td><td>$3.00/ac</td><td>Total Costs</td><td>$0.50/acre</td></tr>
<tr><td colspan="3">Total Benefits – Total Costs = Net Gain</td><td>$2.50/acre</td></tr>
</table>

<u>Assumptions</u>

- Corn Price $3.00/bu
- N price $0.50/lb

Partial budget analysis for each addition of N fertilizer

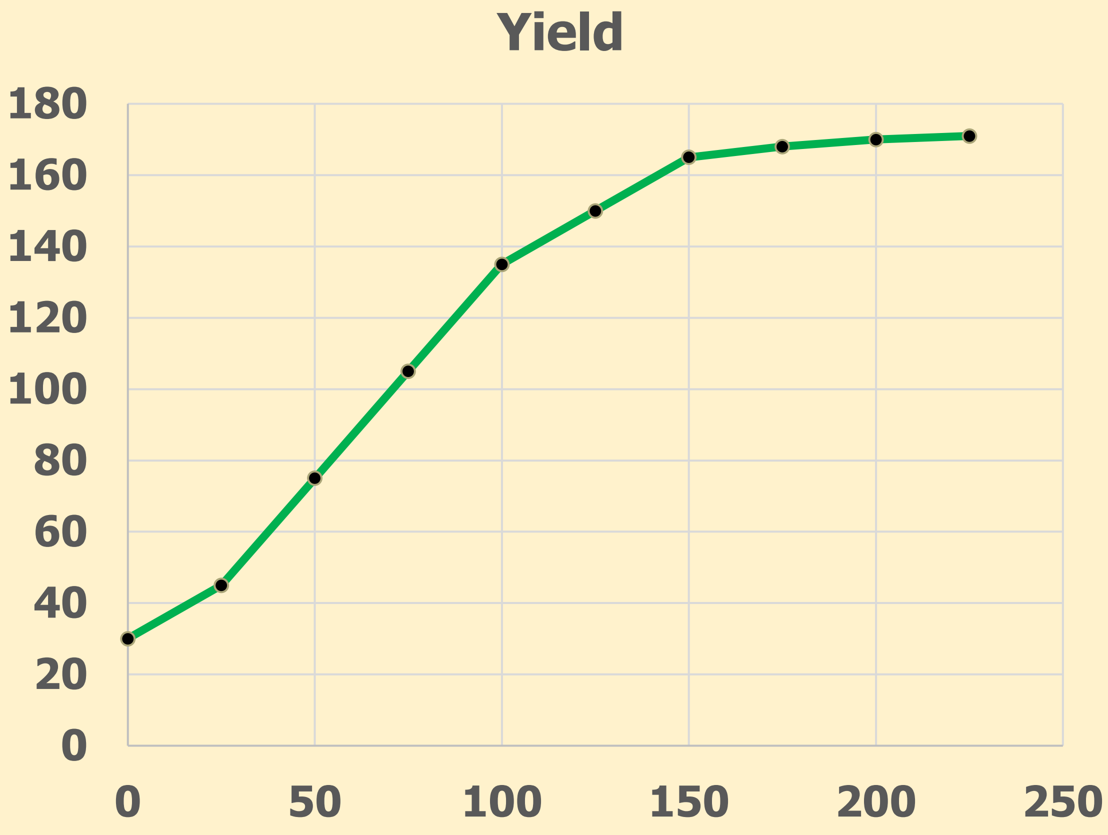
- Do I add 25 more lbs of N fertilizer?
- Corn price = \$2.50/bu
- N price = \$0.30/lb

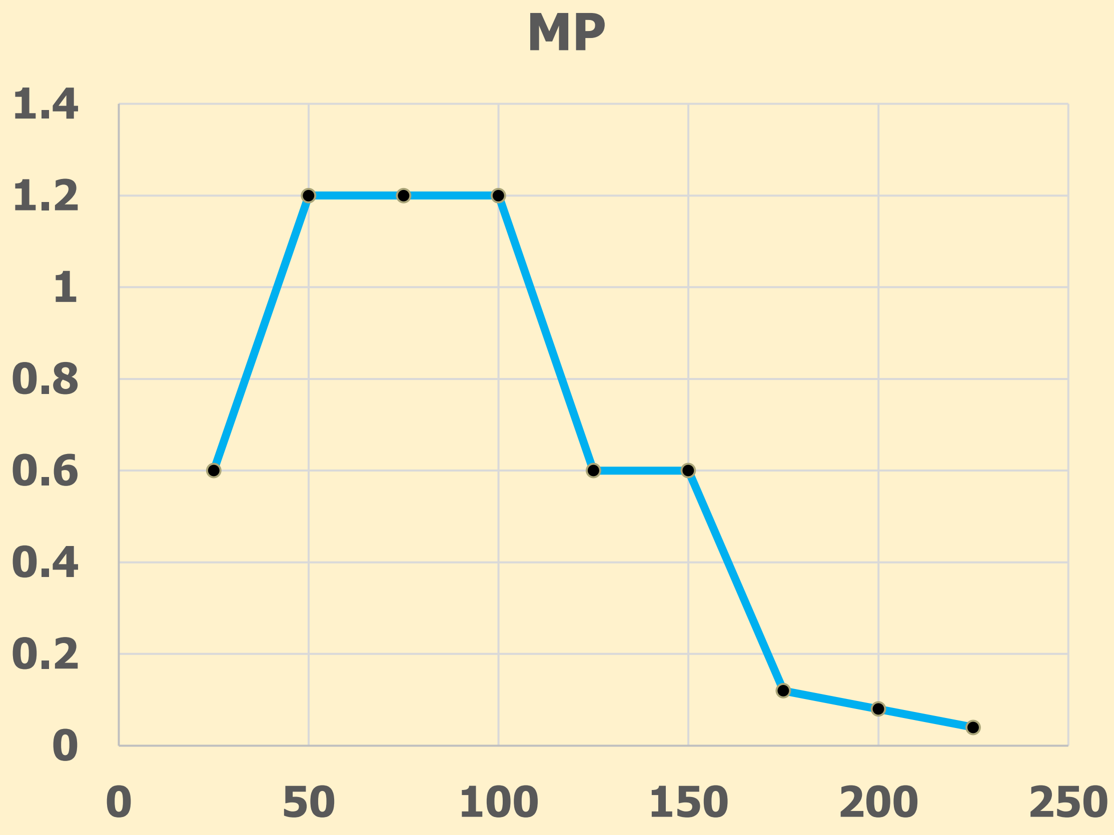
		\$2.50 x ΔQ	\$0.30 x ΔN
N lbs/ac	Yield bu/ac	Benefit of Extra N	Cost of Extra N
0	30	---	---
25	45	\$37.50	\$7.50
50	75	\$75.00	\$7.50
75	105	\$75.00	\$7.50
100	135	\$75.00	\$7.50
125	150	\$37.50	\$7.50
150	165	\$37.50	\$7.50
175	168	\$7.50	\$7.50
200	170	\$5.00	\$7.50
225	171	\$2.50	\$7.50

- What if you only wanted to add **one** more pound of N?
- Let's divide the Benefit and Cost by the ΔN of 25 pounds
- Gives the net gain for 1 extra pound of N

		\$2.50 x ΔQ	ΔQ/ΔN	\$2.50 x ΔQ/ΔN	\$0.30 x ΔN	\$0.30 x ΔN/ΔN
N	Yield	Benefit of Extra N	MP	Benefit of 1 extra lb of N	Cost of Extra N	Cost of 1 extra lb of N
0	30	---	---	---	---	---
25	45	\$37.50	0.6	\$1.50	\$7.50	\$0.30
50	75	\$75.00	1.2	\$3.00	\$7.50	\$0.30
75	105	\$75.00	1.2	\$3.00	\$7.50	\$0.30
100	135	\$75.00	1.2	\$3.00	\$7.50	\$0.30
125	150	\$37.50	0.6	\$1.50	\$7.50	\$0.30
150	165	\$37.50	0.6	\$1.50	\$7.50	\$0.30
175	168	\$7.50	0.12	\$0.30	\$7.50	\$0.30
200	170	\$5.00	0.08	\$0.02	\$7.50	\$0.30
225	171	\$2.50	0.04	\$0.01	\$7.50	\$0.30

Graphics


Yield
180
160
140
120
100
80
60
40
20
0
0
50
100
150
200
250


MP
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250

Intuition

- If you take a partial budget analysis and divide by ΔX (the change in X in the table), you convert it to a partial budget analysis of a 1 unit change in the input X
- The Benefit will be the Marginal Product times the output price, or the Value of the Marginal Product (VMP)
- The Cost will be the cost of 1 unit of X, the Input Price
- How much X to use? Where the VMP = Input Price is the "break even" X if you did a partial budget analysis
  - Keep increasing X until the gain the last bit of input generates just equals the cost of buying the last bit of input

Milk Cow Feed Example

X Feed	Q Milk	MP	VMP	r Feed Price	π profit
0	0			$180	-$400
1	800	800	$144	$180	-$436
2	1,700	900	$162	$180	-$454
3	3,000	1300	$234	$180	-$400
4	5,000	2000	$360	$180	-$220
5	8,500	3500	$630	$180	$230
6	11,700	3200	$576	$180	$626
7	14,600	2900	$522	$180	$968
8	17,200	2600	$468	$180	$1,256
9	19,400	2200	$396	$180	$1,472
10	21,000	1600	$288	$180	$1,580
11	22,000	1000	$180	$180	$1,580
12	22,600	600	$108	$180	$1,508
13	22,800	200	$36	$180	$1,364
14	22,900	100	$18	$180	$1,202

Milk Price = $18/cwt
or p = $0.18/lb

Feed Price = $180 for 1,000 lbs

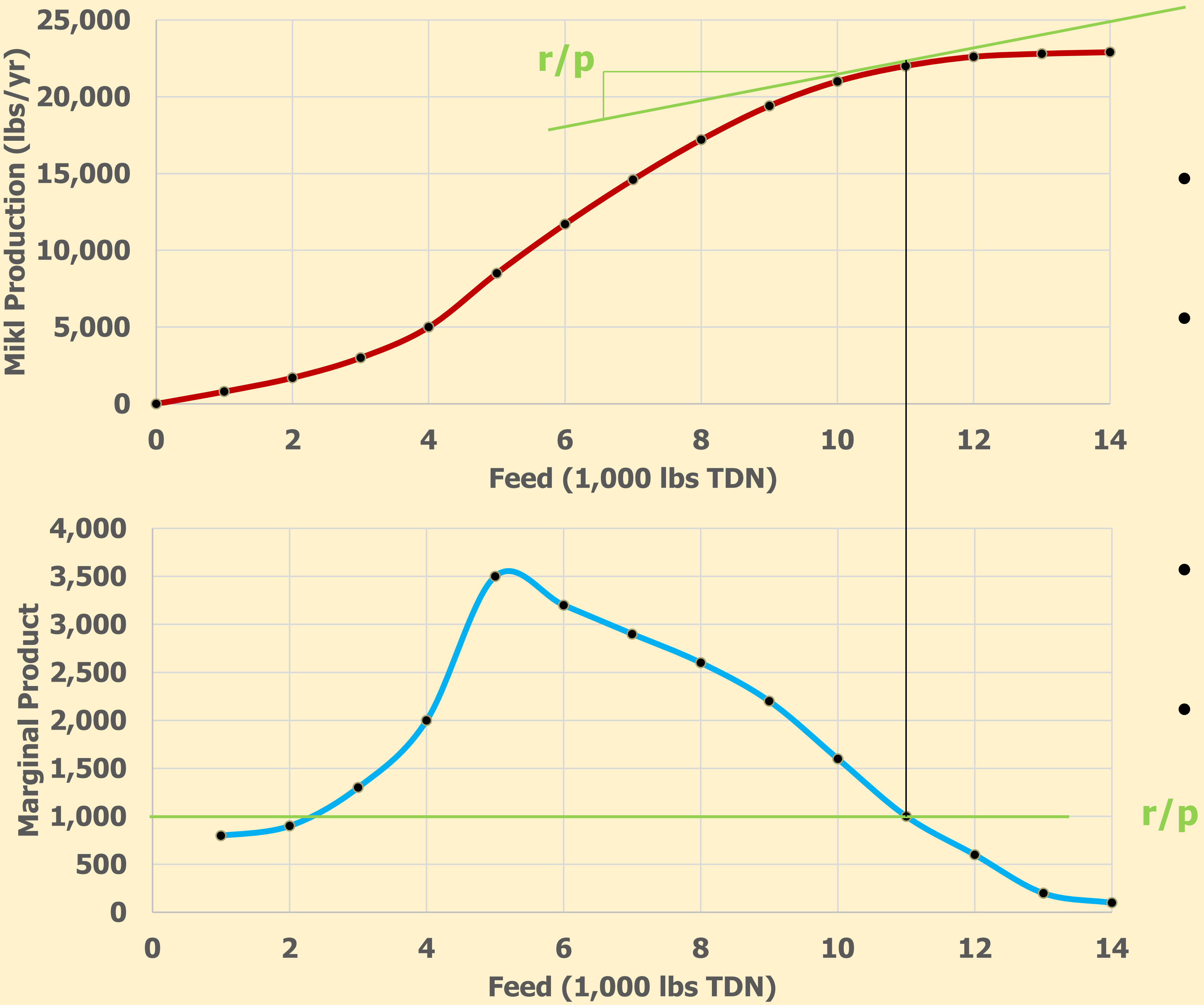
Fixed Cost = $400/yr

Price Ratio r/p = $180/$0.18 = 1,000

VMP = r at Feed = 11

Optimal Feed = 11

MP = r/p = 1,000



- Profit Max occurs where VMP = r
- Where line with slope of r/p = MP, or is tangent to the production function
- r/p = 180/0.18 = 1,000
- Where MP = r/p line

Another Way to Find Optimal Input Use

- Have derived the profit maximizing condition defining optimal input use

  $p \times MP = r$ or $VMP = r$
- Rearrange this condition to get an alternative: $MP = r/p$
- Find the amount of X where the MP equals r/p
- r/p is the "Relative Price" of input X, how much X is worth in the market relative to Q
- r is $ per unit of X, p is $ per unit of Q
- Ratio r/p is units of Q per one unit of X
- r/p is how much Q you could buy if you traded in one unit of X
- r/p is the cost of X if you were buying X in the market using Q in trade

MP = r/p and Cow Feed

- r = $/ton of TDN (Feed), p = $/cwt of milk, so
- r/p = ($/ton)/($/cwt) = cwt/ton, or the hundredweight of milk you could buy if you "traded in" one ton of feed
- MP = cwt of milk from the last ton of feed
- MP = r/p means to find the amount of Feed that gives the same conversion between Feed and milk in the production process as in the market, or find the Feed amount that sets the **Marginal Benefit of Feed = Marginal Cost of Feed**

Milk Cow Feed Example: Key Points

- Profit maximizing Feed is less than output maximizing Feed, which implies profit maximization ≠ output maximization
- Profit maximizing Feed occurs at Feed levels where MP is decreasing, meaning that you use Feed where it has a diminishing MP
- Profit maximizing Feed depends on both the Feed price and the milk price
- Profit maximizing Feed same whether use VMP = r or MP = r/p to identify optimal Feed

Think Break #3

- Fill in the VMP column in the table using $3/bu for the corn price.
- What is the profit maximizing N fertilizer rate if the N fertilizer price is $0.5/lb?

N lbs/A	Yield bu/A	MP	VMP
0	30	---	---
25	45	0.6	
50	75	1.2	
75	105	1.2	
100	135	1.2	
125	150	0.6	
150	165	0.6	
175	168	0.12	
200	170	0.08	
225	171	0.04	

Why We Need Calculus

- What do you do if the VMP = r is not in the table?
- If you have the production function Q = f(X), then you can use calculus to derive an equation for the MP = f′(X)
- With an equation for MP, you can “fill in the gaps” in the tabular form of the production schedule

Calculus and AAE 320

- I will keep the calculus simple!!!
- Production Functions will be Quadratic Equations: $Q = f(X) = a + bX + cX^2$
- First derivative = slope of production function = Marginal Product
- 3 different notations for derivatives
- dy/dx (Newton), $f'(x)$ and $f_x(x)$ (Leibniz)
- 2nd derivatives: d^2y/dx^2, $f''(x)$, $f_{xx}(x)$

Quick Review of Derivatives

- Constant Function
  - If $Q = f(X) = K$, then $f'(X) = 0$
  - $Q = f(X) = 7$, then $f'(X) = 0$
- Power Function
  - If $Q = f(X) = aX^b$, then $f'(X) = abX^{b-1}$
  - $Q = f(X) = 7X = 7X^1$, then $f'(X) = 7(1)X^{1-1} = 7$
  - $Q = f(X) = 3X^2$, then $f'(X) = 3(2)X^{2-1} = 6X$
- Sum of Functions
  - $Q = f(X) + g(X)$, then $dQ/dX = f'(X) + g'(X)$
  - $Q = 3 + 5X - 0.1X^2$, $dQ/dX = 0 + 5 - 0.2X$

Think Break #4

- What are the 1st and 2nd derivatives with respect to X of the following functions?
  1. $Q(X) = 4 + 15X - 7X^2$
  2. $\pi(X) = 2(5 - X - 3X^2) - 8X - 15$

Calculus-Based Approach to Optimal Input Use

Mathematical Model: Profit = Revenue – Cost

Profit = price x output – input cost – fixed cost

$\pi = pQ - rX - K = \mathbf{pf(X) - rX - K}$

π = profit   Q = output   X = input

f(X) = production function

p = output price   r = input price   K = fixed cost

- Learn this model, we will use it a lot!!!

Calculus-Based Approach to Optimal Input Use

- Find X to Maximize profit π**(X) = pf(X) – rX – K**
- Calculus: Set first derivative of π with respect to X equal to 0 and solve for X, the "First Order Condition" (FOC)
- FOC: $pf'(X) - r = 0$ $p \times MP - r = 0$
- Rearrange: $pf'(X) = r$ $p \times MP = r$
- p x MP is the "Value of the Marginal Product" (VMP), what would get if sold the MP
- FOC means to increase use of the input X until p x MP = r, or until the VMP = r, the input price

Calculus of Optimization

- Problem: Choose X to Maximize some function g(X)
- First Order Condition (FOC): Set g′(X) = 0 and solve for X
- May be more than one X (not in this class)
- Call these potential solutions X^*
- Identifying X values where the slope of the objective function is zero (satisfies the FOC)
- Use SOC to see if at maximum or minimum

Calculus of Optimization

- Second Order Condition (SOC)
- Evaluate $g''(X)$ at each X^* identified
- Condition for a maximum is $g''(X^*) < 0$
- Condition for a minimum is $g''(X^*) > 0$
- $g''(X)$ is the function's curvature at X
- Positive curvature is convex (minimum)
- Negative curvature is concave (maximum)

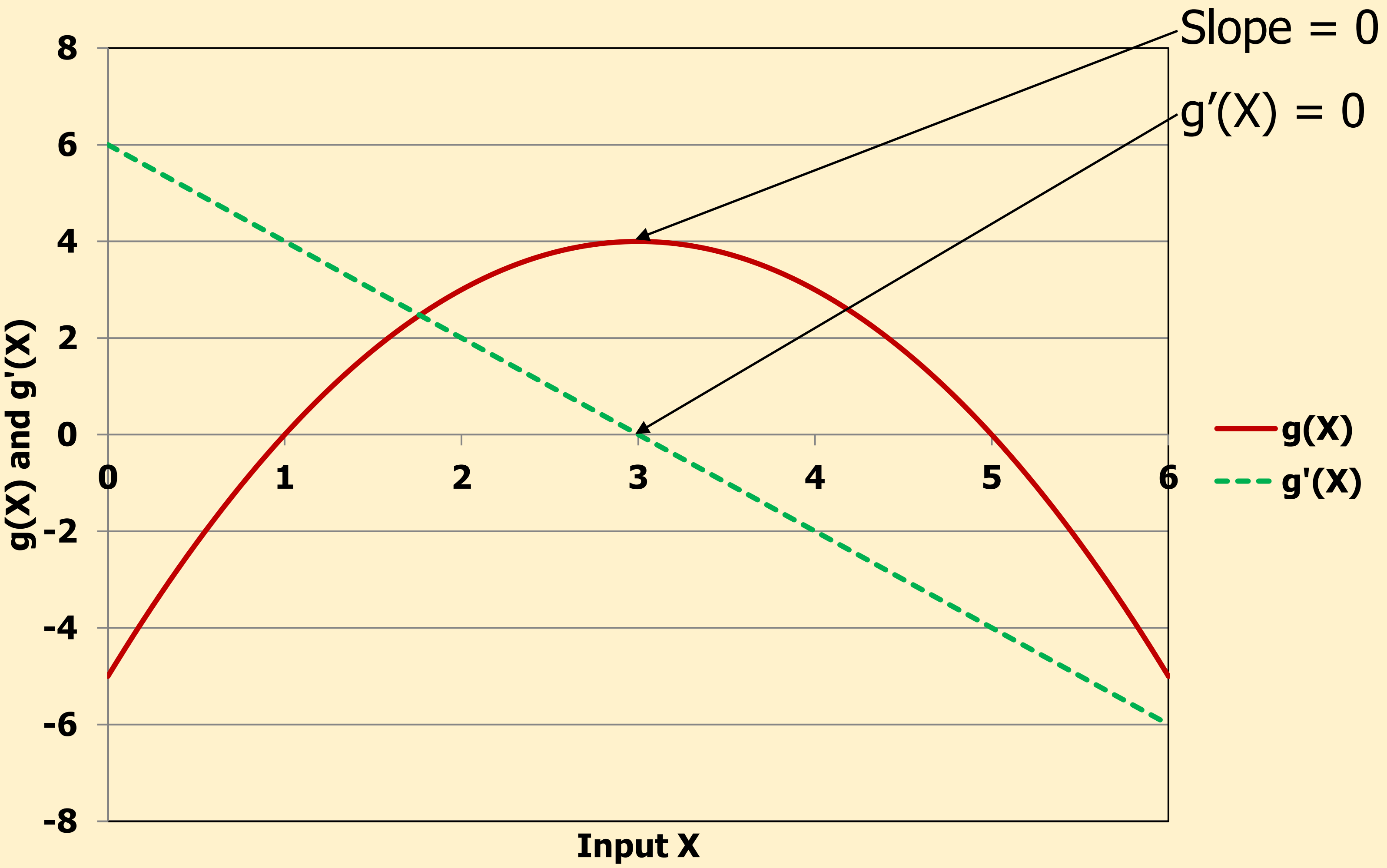
Calculus of Optimization: Intuition

- FOC: finding the X values where the objective function's slope is zero, candidates for minimum/maximum
- SOC: checks curvature at each candidate solution identified by setting FOC equal to zero
- Maximum is curved down (2^{nd} derivative <u>negative</u>)
- Minimum is curved up (2^{nd} derivative <u>positive</u>)

Example 1

- Choose X to maximize $g(X) = -5 + 6X - X^2$
- FOC: $g'(X) = 6 - 2X = 0$
- FOC satisfied when $X = 3$
- Is this a maximum or a minimum or an inflection point? How do you know?
- Check the SOC: $g''(X) = -2 < 0$
- Negative, satisfies SOC for a maximum
- The value of $g(X)$ at $X = 3$: $g(3) = -5 + 6(3) - 3^2 = 4$

Example 1: Graphics

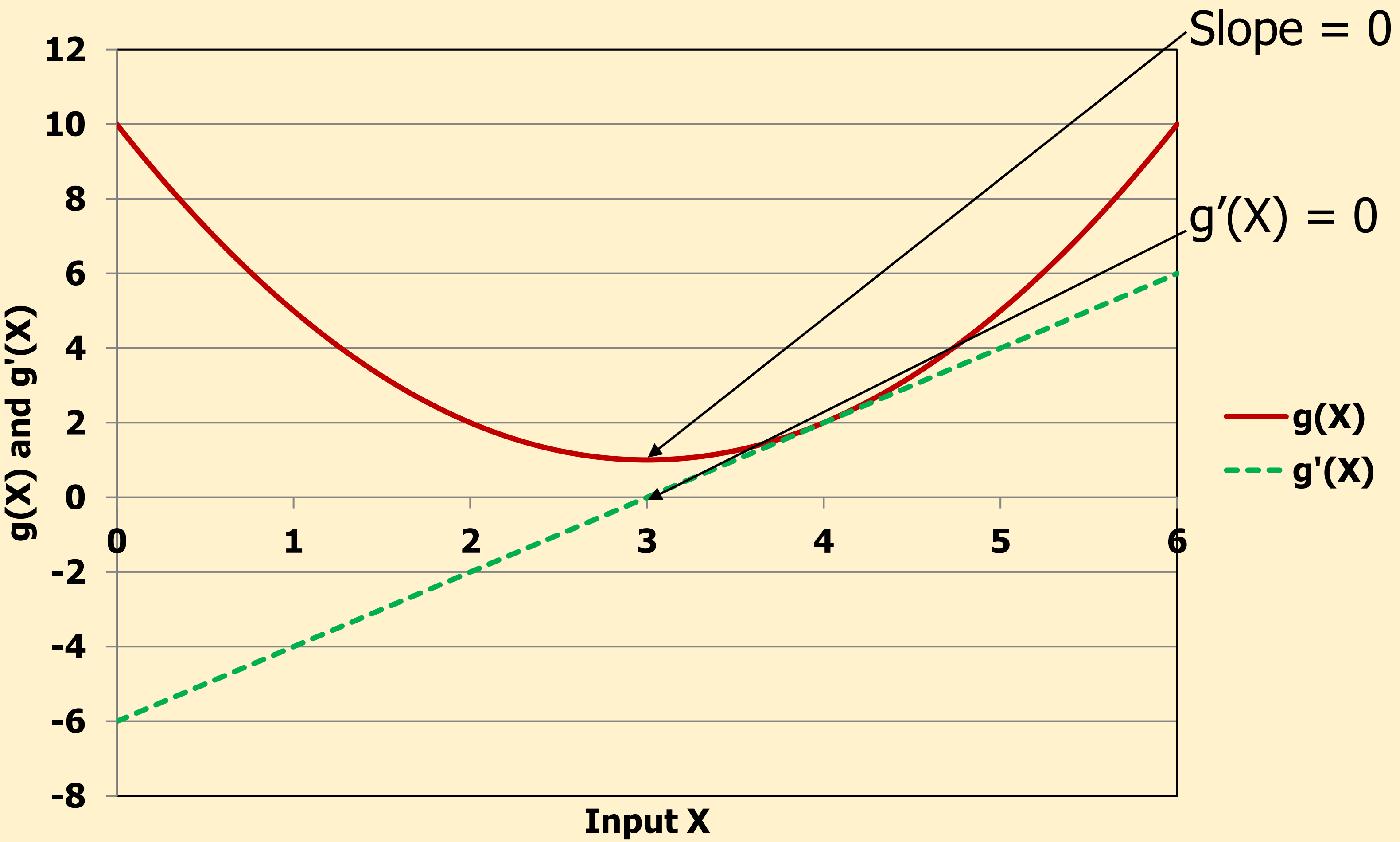


Example 2

- Choose X to maximize $g(X) = 10 - 6X + X^2$
- FOC: $g'(X) = -6 + 2X = 0$
- FOC satisfied when $X = 3$
- Is this a maximum or a minimum or an inflection point? How do you know?
- Check the SOC: $g''(X) = 2 > 0$
- Positive, does not satisfy SOC for maximum
- The value of $g(X)$ at $X = 3$: $g(3) = 10 - 6(3) + 3^2 = 1$

Example 2: Graphics

What value of X maximizes this function?



Think Break #5

Choose X to Maximize:

$\pi(X) = 10(30 + 5X - 0.4X^2) - 2X - 18$

1) What X satisfies the FOC?

2) Does this X satisfy the SOC for a maximum?

3) What is $\pi(X)$ at this X?

Calculus and Production Economics

- In general, $\pi(X) = pf(X) - rX - K$
- Suppose your production function is

  $Q = f(X) = 30 + 5X - 0.4X^2$
- Suppose output price is 10, input price is 2, and fixed cost is 18, then

  $\pi = 10(30 + 5X - 0.4X^2) - 2X - 18$
- To find X to maximize π, solve the FOC and check the SOC, can then calculate output and profit

Calculus and Production Economics

- $\pi = 10(30 + 5X - 0.4X^2) - 2X - 18$
- FOC: $10(5 - 0.8X) - 2 = 0$

$10(5 - 0.8X) = 2$ $p \times MP = r$
$5 - 0.8X = 2/10$ $MP = r/p$

When solving the FOC, you set VMP = r and/or MP = r/p & solve for X

- Solve for X = 6, check SOC, confirm that it is a maximum
- Plug 6 back into the profit function to calculate output Q and then profit π
- $Q(X) = 30 + 5X - 0.4X^2$
- $Q(6) = 45.6$
- $\pi(X) = 10(Q) - 2X - 18$
- $\pi(6) = 10(45.6) - 30 = 426$

Summary: Single Input Production Function

- Condition to find optimal input use:

  VMP = r or MP = r/p
- What does this condition mean?
- What does it look like graphically?
- Know how to use condition to find optimal input use and corresponding output and profit
  - 1) With a production schedule (table)
  - 2) With a production function (calculus)