Multiple Input Production Economics

EXAMPLES

AAE 320
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Equal Margin Principle

Equal Margin Principle: expressed mathematically in two ways

1) \[ \frac{\text{MP}_x}{r_x} = \frac{\text{MP}_y}{r_y} \]  
   Ratio of MP/r must be equal for all inputs

2) \[ \frac{\text{MP}_x}{\text{MP}_y} = \frac{r_x}{r_y} \]  
   Ratio of MP’s must equal input price ratio
Marginal Rate of Technical Substitution

• The ratio of marginal products (MP_x/MP_y) is the substitution rate between inputs in the production process
  • Slope of the tradeoff curve
• MP_x/MP_y is called the Marginal Rate of Technical Substitution (MRTS): the input substitution rate at the margin
• If you cut X by one unit, how much must you increase Y to keep output the same
• If you increase Y by one unit, how much can you cut X and still keep the same output
• Optimality condition MP_x/MP_y = r_x/r_y means set substitution rates equal
Isoquants and $\frac{MP_x}{MP_y}$

- Isoquant Slope = $\frac{\Delta Y}{\Delta X}$ = Substitution rate between X and Y at the margin
- If reduce X by the amount $\Delta X$, then must increase Y by the amount $\Delta Y$ to keep output fixed
- $\frac{dY}{dX} = -\frac{MP_x}{MP_y} = -\text{Ratio of MP's} = -\text{MRTS}$
- Isoquant Slope = $-\text{MRTS}$

Output $Q = \tilde{Q}$
Isoquants and \[ \frac{MP_x}{MP_y} = \frac{r_x}{r_y} \]

- Optimality condition for input use from the Equal Margin Principle: \( MP_x/MP_y = r_x/r_y \)
- Find the input levels for X and Y that set slope of the isoquant = the price ratio
- Point of tangency between price line with slope \(- r_x/r_y\) and isoquant with slope \(\Delta Y/\Delta X = -MP_x/MP_y\)

Output \( Q = \tilde{Q} \)

Slope = \(- r_x/r_y\)
<table>
<thead>
<tr>
<th>Soybean Meal (lbs)</th>
<th>Corn (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>376.8</td>
</tr>
<tr>
<td>15</td>
<td>356.3</td>
</tr>
<tr>
<td>20</td>
<td>339.0</td>
</tr>
<tr>
<td>25</td>
<td>326.0</td>
</tr>
<tr>
<td>30</td>
<td>315.0</td>
</tr>
<tr>
<td>35</td>
<td>307.5</td>
</tr>
<tr>
<td>40</td>
<td>300.6</td>
</tr>
<tr>
<td>45</td>
<td>294.6</td>
</tr>
<tr>
<td>50</td>
<td>289.2</td>
</tr>
<tr>
<td>55</td>
<td>284.4</td>
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<tr>
<td>60</td>
<td>280.2</td>
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<tr>
<td>65</td>
<td>276.4</td>
</tr>
<tr>
<td>70</td>
<td>272.9</td>
</tr>
</tbody>
</table>

Soybean meal and corn needed for 125 lb feeder pigs to gain 125 lbs

Which feed ration do you use?
<table>
<thead>
<tr>
<th>Soy Meal (lbs)</th>
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<th>P ratio</th>
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<tbody>
<tr>
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<tr>
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<td>2.20</td>
</tr>
<tr>
<td>20</td>
<td>339.0</td>
<td>3.46</td>
<td>2.20</td>
</tr>
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<td>326.0</td>
<td>2.60</td>
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<td>1.20</td>
<td>2.20</td>
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<tr>
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<td>289.2</td>
<td>1.08</td>
<td>2.20</td>
</tr>
<tr>
<td>55</td>
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<td>0.96</td>
<td>2.20</td>
</tr>
<tr>
<td>60</td>
<td>280.2</td>
<td>0.84</td>
<td>2.20</td>
</tr>
<tr>
<td>65</td>
<td>276.4</td>
<td>0.76</td>
<td>2.20</td>
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<td>272.9</td>
<td>0.70</td>
<td>2.20</td>
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- Economically Optimal input use is where MRTS = input price ratio, $\Delta Y/\Delta X = r_x/r_y$
- (Same as $MP_x/MP_y = r_x/r_y$)
- Soy Meal Price = $0.088/lb, Corn Price = $0.04/lb, Ratio = 0.088/0.04 = 2.20
- Keep straight which is X and which is Y!!!

\[
1.20 = - \frac{(294.6 - 300.6)}{(45 - 40)}
\]

\[
0.96 = - \frac{(284.4 - 289.2)}{(55 - 50)}
\]
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<tr>
<td>70</td>
<td>272.9</td>
<td>0.70</td>
<td>2.20</td>
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</tbody>
</table>

- Suppose Soy Meal Price = $0.10/lb, Corn Price = $0.05/lb, Ratio = 0.10/0.05 = 2.0

- Interpolation

<table>
<thead>
<tr>
<th>B btwn Unknown</th>
<th>A btwn Known</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soy</td>
<td>Corn</td>
</tr>
<tr>
<td>Row 0</td>
<td>30</td>
</tr>
<tr>
<td>Between</td>
<td></td>
</tr>
<tr>
<td>Row 1</td>
<td>40</td>
</tr>
</tbody>
</table>

\[
B_{btw} = B_0 + (A_{btw} - A_0) \frac{B_1 - B_0}{A_1 - A_0}
\]

\[
B_{btw} = 30 + (2.0 - 2.2) \frac{40 - 30}{1.5 - 2.2} = 32.9
\]

\[
B_{btw} = 315 + (2.0 - 2.2) \frac{300.6 - 315}{1.5 - 2.2} = 310.9
\]
## Example 1

The table is rations of grain and hay that put 300 lbs of gain on 900 lb steers.

1) Fill in the missing MRTS

2) If grain is $0.06/lb and hay is $0.03/lb, what is the economically optimal feed ration?

<table>
<thead>
<tr>
<th>Grain (lbs)</th>
<th>Hay (lbs)</th>
<th>MRTS</th>
<th>Price ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>825</td>
<td>1350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>1130</td>
<td>2.93</td>
<td></td>
</tr>
<tr>
<td>975</td>
<td>935</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1050</td>
<td>770</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td>1125</td>
<td>625</td>
<td>1.93</td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td>525</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1275</td>
<td>445</td>
<td>1.07</td>
<td></td>
</tr>
</tbody>
</table>
Types of Input Substitution

- **Perfect substitutes**: soybean meal & canola meal, corn & sorghum, wheat and barley
- **Imperfect substitutes**: corn & soybean meal
- **Non-substitutes/perfect complements**: tractors and drivers, wire and fence posts
Perfect Substitutes: Graphics

Use only Canola Meal
\[ \frac{r_x}{r_y} > 2 \]

Use any combination of Canola or Soybean Meal
\[ \frac{r_x}{r_y} = 2 \]

Use only Soybean Meal
\[ \frac{r_x}{r_y} < 2 \]
Economics of Imperfect Substitutes

This the case we already did!
Perfect Complements: Graphics

Price ratio does not matter
Example 2

• Corn yield is \( Y = 10 + 12N - 0.2N^2 + 15W - 0.3W^2 - 0.1NW \),
• \( Y \) = corn yield (bu/ac), \( N \) = nitrogen (lbs/ac), \( W \) = water (acre inches)
• Corn price = $3/bu, \( N \) price = $0.5/lb, \( W \) price = $12/acre inch
• What is the profit maximizing amount of nitrogen (\( N \)) and water (\( W \)) to use per acre to grow corn (\( Y \))?
• Set Up:
  • FOCs:
  • SOCs:
• Yield & Profit at optimal \( N \) and \( W \):