Single Input Production Economics for Farm Management

EXAMPLES

AAE 320: Farming Systems Management

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Example #1

- Fill in the columns in the table using $3.50/bu for the corn price
- What is the profit maximizing N fertilizer rate if the N fertilizer price is $0.35/lb?

N lbs/A	Yield bu/A	AP	MP	VMP
0	50	---	---	---
20	90		2	$7
40	120			
60	148			
80	172			
100	192			
120	210			
140	222			
160	228			
180	230			

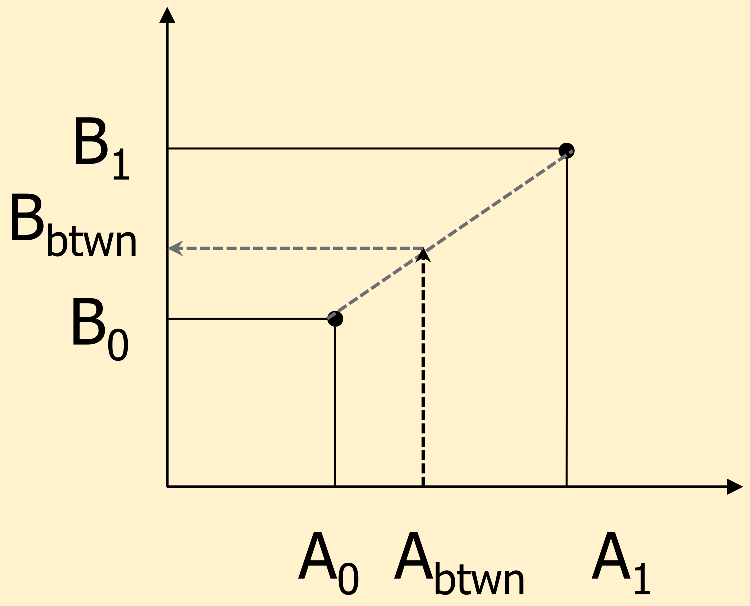
Example #1 Answer

N lbs/A	Yield bu/A	AP	MP	VMP
0	50	---	---	---
20	90	90/20=4.50	(90-50)/(20-0)=2	2x3.50=7.00
40	120	120/40=3.00	(120-90)/(40-20)=1.5	1.5x3.50=5.25
60	148	148/60=2.47	(148-120)/(60-40)=1.4	1.4x3.50=4.90
80	172	172/80=2.15	(172-148)/(80-60)=1.2	1.2x3.50=4.20
100	192	1.92	1	3.50
120	210	1.75	0.9	3.15
140	222	1.59	0.6	2.10
160	228	1.43	(228-222)/(160-140)=0.3	1.05
180	230	1.28	(230-228)/(180-160)=0.1	0.35

Linear Interpolation

- You have a variable A with two values A_0 and A_1 and <u>matched</u> with it is another variable B with two values B_0 and B_1
- You know A_{btwn} that is between A_0 and A_1 and you want to linearly interpolate what the corresponding B_{btwn} is

How do you find B_{btwn} when you know A_0, A_1, B_0, B_1 and A_{btwn}?



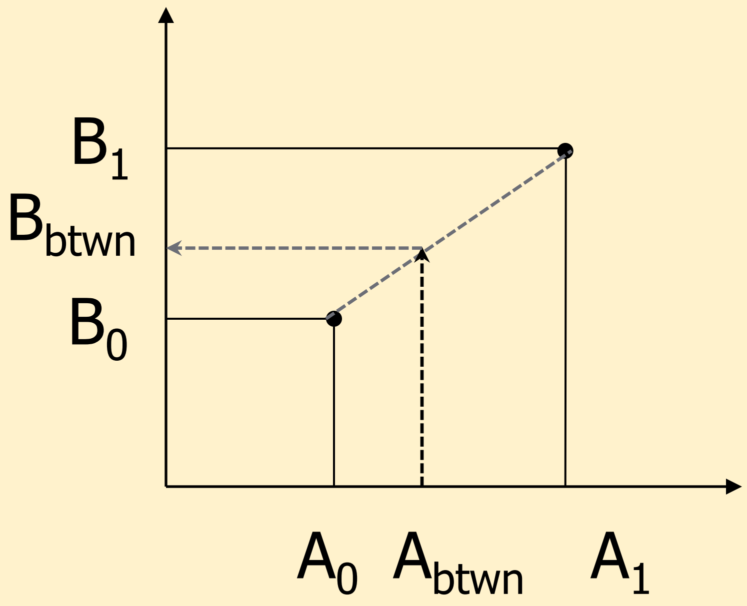
Linear Interpolation

- This formula will hold

$$\frac{B_{btwn} - B_0}{A_{btwn} - A_0} = \frac{B_1 - B_0}{A_1 - A_0}$$

- Solve this for B_{btwn}

$$\boldsymbol{B_{btwn} = B_0 + (A_{btwn} - A_0)\frac{B_1 - B_0}{A_1 - A_0}}$$



Linear Interpolation Example

- Suppose have inputs & VMPs from this table & input price is $0.60
- What is the optimal N, linearly interpolating between 1.05 & 0.35?
- $B_{btwn} = B_0 + (A_{btwn} - A_0)\frac{B_1 - B_0}{A_1 - A_0}$
- $B_{btwn} = 160 + (0.6 - 1.05)\frac{180 - 160}{0.35 - 1.05}$
- $B_{btwn} = 160 + (-0.45)\frac{20}{-0.7} =$ **172.9**

	B_{btwn} Unknown Input N (B)	Yield	MP	A_{btwn} Known VMP (A)
Row 0	160	228	0.3	1.05
Row 1	180	230	0.1	0.35

Example #2

- You manage a small vegetable farm, the table reports how many bags of potatoes are dug, cleaned, and ready for sale in one hour with different numbers of workers
- How many workers is it optimal to hire if potatoes sell for $2/bag and you hire workers for $20/hr? What if the wage is $18/hr?

Workers	Bags/hr	MP	VMP
3	50		
5	75		
7	95		
9	105		

Example #2 Answer

- How many workers is it optimal to hire if potatoes sell for \$2/bag and you hire workers for \$20/hr? What if the wage is \$18/hr?
- MP = (75-50)/(5-3) = 12.5 VMP = 2 x MP = \$25
- Optimal where VMP = r occurs at 7 workers where VMP = \$20

Workers	Bags/hr	MP	VMP
3	50	--	--
5	75	12.5	25
7	95	10	20
9	105	5	10

Interpolation for wage = \$18/hr

- $Y_{btwn} = Y_0 + (X_{btwn} - X_0)\frac{Y_1 - Y_0}{X_1 - X_0}$
- $Y_{btwn} = 7 + (18 - 20)\frac{7-9}{20-10}$
- $Y_{btwn} = \mathbf{7.4}$

Example #3

- Soybean yield is $Q = 3 + 2S - 0.01S^2$, where yield Q is total bushels per acre (bu/ac) and the seeding rate S is thousands of seeds planted per acre. The soybean price is $9/bu and seeds cost $0.35 per thousand seeds.
- What is the economically optimal seeding rate (S) to plant? Set up and solve this economic problem using calculus. Check the second order condition.
- At the seeding rate S you find, what is the soybean yield (bu/ac)?
- Besides seed costs, other costs = $800/ac. What are net returns ($/ac)?

Example #3: Answer

- Set Up: $\pi = 9(3 + 2S - 0.01S^2) - 0.35S$
- FOC: $d\pi/dS = 9(2 - 0.02S) - 0.35 = 0$
- Solve FOC: $9(2 - 0.02S) = 0.35$
  - $2 - 0.02S = 0.35/9 = 0.039$ $\quad$ $2 - 0.039 = 0.02S$
  - $S = 1.961 / 0.02 = 98.05$ (or 98 thousand seeds/ac)
- SOC: $d^2\pi/dS^2 = 9(-0.02) = -1.8 < 0$ passes SOC for maximum
- Yield: $Q = 3 + 2S - 0.01S^2 = 3 + 2(98) - 0.01(98)^2 = 102.96 = 103$ bu/ac
- Profit: $\pi = 9(103) - 0.35(98) - 800 = \$92.70$/ac
- Profit: $\pi = 9(102.96) - 0.35(98.05) - 800 = \$92.32$/ac

Example #4

- Corn yield is $Y = 150 + 0.994N - 0.00228N^2$, where Y is corn yield in bushels per acre and N is the total nitrogen applied as pounds of N per acre. The corn price is $5/bu. Urea N nitrogen fertilizer solution (which is 45% N by weight) costs $400/ton.
- What nitrogen application rate maximizes net returns? Set up and solve this economic problem using calculus. Check the second order condition.
- At the nitrogen rate N you find, what is the corn yield (bu/ac)?
- Besides nitrogen costs, other costs = $900/ac. What are net returns ($/ac)?

Example #4: Price Conversions

- Many ag inputs often require price conversions. For example, fertilizer is priced in dollars per ton and often needs to be converted to dollars per pound of the active ingredient (Nitrogen).
- You buy ammonium nitrate (NH_4NO_3), which is 34% N, for $250 per ton, what is the cost of the Nitrogen in $ per pound?
- $\frac{\$300}{ton\ of\ NH4NO3}$ x $\frac{1\ ton\ of\ NH4NO3}{0.34\ ton\ of\ N}$ x $\frac{1\ ton\ of\ N}{2{,}000\ lbs\ of\ N}$ =$0.44/lb of N
- Why 34%? Atomic masses N=14, H=1, O=16: 2 N's divided by the mass of the molecule: 28/(28+4+48) = 24/80 = 35%

Example #4: Answer

- Set Up: $\pi = 5(150 + 0.994N - 0.00228N^2) - 0.44N$
  - Price conversion $\frac{\$400}{ton\ of\ Urea}$ x $\frac{1\ ton\ of\ Urea}{0.45\ ton\ of\ N}$ x $\frac{1\ ton\ of\ N}{2{,}000\ lbs\ of\ N}$ =\$0.44/lb of N
- FOC: $d\pi/dN = 5(0.994 - 0.00456N) - 0.44 = 0$
- Solve FOC: $5(0.994 - 0.00456N) = 0.44$
  - $0.994 - 0.00456N = 0.44/5 = 0.088$ $\qquad 0.994 - 0.088 = 0.00456N$
  - $N = 0.906 / 0.00456 = 198.7$ lbs/ac
- SOC: $d^2\pi/dN^2 = 5(-0.\ 0.00456) = -0.0228 < 0$ passes SOC for maximum
- Yield: $Y = 150 + 0.994(199) - 0.00228(199)^2 = 257.5$ bu/ac
- Profit: $\pi = 5(257.5) - 0.44(199) - 900 = \$299.94/ac = \$300/ac$