1) You are a crop consultant helping a grain farmer in Wisconsin’s Central Sands determine his economically optimal irrigation level. The last few years you ran on-farm irrigation experiments to create the first two columns in the table below for his average yield for different levels of total water (rainfall plus irrigation) during the growing season.

Fill in the table below and answer the following questions.

<table>
<thead>
<tr>
<th>Total Water (acre inches)</th>
<th>Average Corn Yield (bu/ac)</th>
<th>Marginal Product (bu/ac)</th>
<th>Value of the Marginal Product ($/ac)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>265.0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>14</td>
<td>273.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>280.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>287.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>293.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>298.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>302.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>304.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When he plants, he forward contracts to sell his corn at a price of $3.00/bu. You estimate that his cost of pumping water is $9.00 per acre inch.

a) Suppose your client typically makes sure his corn gets 22 inches of water and this year he wants to go with 24 inches. Answer the following questions in order to explain to him why this is not a good plan if he wants to maximize his net returns:
   i. When he goes from 22 to 24 inches, how much does his average yield increase?
   ii. How much is this extra yield worth?
   iii. How much will it cost him to pump these extra 2 inches of irrigation water?
   iv. Is adding these two inches of water a good idea if he wants to maximize his net returns? Why or why not?
b) Assume he wants to maximize his net returns.
   i. How much water should he plan make sure his corn has (you may need to interpolate)?

   ii. At this amount of water, what is his average corn yield and what is its value at $3.00/bu?

   iii. If it usually rains 12 inches during the growing season, how much water will he have to
       pump? What will it cost to pump this much water if pumping costs $9.00 per acre inch?

   iv. Assuming he has $600/ac in other costs, what are his net returns per acre?

c) Suppose he replaces his current high-pressure irrigation system with a more efficient low-
   pressure system that uses less energy and wastes less water, and as a result, pumping costs
   decrease to $5.40 per acre inch. Assume he still wants to maximize his net returns.
   How much water should he plan on making sure his corn has?

d) Suppose a group wants to reduce agricultural use of water in the area and to do so, they are
   offering farmers subsidies to replace older high-pressure irrigation systems with new more
   efficient low-pressure systems that use less energy and waste less water and so have lower
   pumping costs per acre-inch. (In other words, the older systems are like in parts 1a and 1b
   and the newer systems are like in part 1c.) Their logic for these subsidies is that with these
   new more energy efficient systems, farmers will pump less water per acre. These new
   systems are more expensive to buy and install than a conventional system, but once in place,
   the new system is more energy efficient and so has a lower cost to pump one acre inch of
   water. Comment on how effective you think this program will be at reducing water use for
   crop production – How does economically optimal input use respond to a lower input price?
2) You are the same crop consultant, but now determining the economically optimal nitrogen rate for your client’s corn fields using Extension nitrogen fertilizer recommendations. You will first derive the optimal rate using calculus, and then using an online tool.

Suppose expected corn yield is \( Y = 165 + 0.996N - 0.0023N^2 \), where \( Y \) is corn yield in bushels per acre and \( N \) is the total nitrogen applied as pounds of N per acre. Assume the expected corn price is $3.20/bu and Urea N nitrogen fertilizer solution (45% N by weight) costs $400/ton.

a) The production function uses nitrogen in pounds of N per acre, but the price is $400 per ton of fertilizer. Convert the price of the fertilizer from $400/ton of fertilizer to $/lb of N. If urea N nitrogen fertilizer solution (45% N by weight) costs $400/ton, what is the price of nitrogen N in $ per pound?

b) Using the price for N, a corn price of $3.20/bu, and the given production function, write the profit (\( \pi \)) equation for the farmer’s net return per acre.

c) Using calculus, what N rate maximizes profit for the farmer? Give the first order condition, solve it for the optimal N rate, and then check the second order condition.

d) At this nitrogen rate, what is his yield (bu/ac)? What is his per acre revenue? How much will it cost ($/ac) to buy this much nitrogen fertilizer? If he has $700/ac in other costs, what are his per acre net returns (profit) at this nitrogen rate?
e) Assume you find some cheaper urea fertilizer for a price of $360/ton, and the price of corn is now $3.30/bu. Using the same production function, what is the optimal nitrogen rate to apply? What is the expected yield at this nitrogen rate? Again, assuming $700/ac in other costs, what are his per acre returns at this nitrogen rate?

f) Many Midwestern states use this method to develop Extension recommendations for nitrogen fertilizer use in corn. Go to http://cnrc.agron.iastate.edu/ for the Corn Nitrogen Rate Calculator and read the “About” tab, especially “Definitions.” Next, choose “Single Price” and then choose “Wisconsin” for the state, “Corn following Soybean” for the rotation, and “Irr. Sands” for the soil type (irrigated sands). Choose the right form of nitrogen “Urea (45% N)” and enter the nitrogen and corn prices from part a and hit the “Calculate” button.

i) Does the price of N in $/lb match your answer in part a? You should answer “Yes” here, if not, go back and get the price right in part a or enter the correct inputs here. Note that the program rounds to the nearest full cent per pound.

ii) At a urea N fertilizer price of $400/ton and a corn price of $3.20/bu, what does the tool indicate is the optimal N rate?

iii) If the Urea N nitrogen fertilizer price is $360/ton and the corn price is $3.30/bu, what does the tool indicate is the optimal N rate?

iv) How do these compare to your answers in parts a and e? They should be very close to the same amount; if not, go back and get things right.

The UW Extension Nutrient and Pest Management Program (http://ipcm.wisc.edu/) has developed free apps for iPhones/iPads and Android Smartphones/Tablets, the “Corn N Rate Calculator” and the “N Price Calculator”. Search in the iTunes Store or Google Play (or see http://ipcm.wisc.edu/apps/) and download them if you want to. When you do, you will see lots of other interesting apps available as well.