AAE 320 Problem Set #2   **Name:**_____KEY_____

1) You are a beef farmer deciding on your ration for steers on your feedlot. You used your records and data from a local consultant to estimate the amount of hay and grain to put 500 pounds of gain on your cattle. You have constructed the table below. Fill in the table below and answer the following questions.

Table for part a when treating hay as input X and grain as input Y

Hay (X) (lbs/animal)	Grain (Y) (lbs/animal)	Marginal Rate of Technical Substitution	Price Ratio	
			Part a	Part b
2,680	900			
2,360	1,070	0.531	1.111	1.313
2,150	1,190	0.571	1.111	1.313
1,950	1,350	0.800	1.111	1.313
1,825	1,490	1.120 ⬅	1.111	1.313
1,665	1,700	➡1.313	1.111	1.313
1,560	1,880	1.714	1.111	1.313

Table for part a when treating hay as input Y and grain as input X

> You can do either version of table.

Hay (Y) (lbs/animal)	Grain (X) (lbs/animal)	Marginal Rate of Technical Substitution	Price Ratio	
			Part a	Part b
2,680	900			
2,360	1,070	1.882	0.900	0.762
2,150	1,190	1.750	0.900	0.762
1,950	1,350	1.250	0.900	0.762
1,825	1,490	0.893 ⬅	0.900	0.762
1,665	1,700	➡0.762	0.900	0.762
1,560	1,880	0.583	0.900	0.762

a) If the price of grain is $180.00/ton and the price of hay is $200.00/ton, what is the economically optimal amount of grain and hay to feed to your cattle? (You may need to interpolate between entries on the table.)

For the 1st table (hay as X and grain as Y)
  First MRTS entry as an example: $-\Delta Y/\Delta X = -(1070 - 900)/(2360 - 2680) = 0.531$
  Price ratio is $r_x/r_y = 200/180 = 1.111$

For the 2nd table (grain as X and hay as Y)
  First MRTS entry as an example: $-\Delta Y/\Delta X = -(2360 - 2680)/(1070 - 900) = 1.882$
  Price ratio is $r_x/r_y = 180/200 = 0.900$

*Optimal input use is where MRTS = price ratio. The price ratio is 1.120 at hay of 1,825 lbs and grain of 1,490 lbs, which is very close to the price ratio of 1.111, and so the optimum as marked by the **maroon** arrow and box in both tables.*

b) How does your answer change if the price of grain falls to $160.00/ton and the price of hay increases to $210/ton? (You may need to interpolate between entries on the table.)

For the 1st table: Price ratio is $r_x/r_y = 210/160 = 1.313$
For the 2nd table: Price ratio is $r_x/r_y = 160/210 = 0.762$
This price ratio equals the MRTS entry for 1,665 lbs of hay and 1,700 lbs of grain, which is the optimum as marked by the light green arrow and box in both tables.

1

2) You have a summer internship with a swine company. Their scientists give you the following production function for the final weight for feeder pigs bought to feed and sell as hogs:

$$W = -585 + 4C + 6S - 0.05C^2 - 0.06S^2 + 0.08CS,$$

where W is each hog's final weight in <u>pounds</u>, C is <u>pounds</u> of ground corn consumed and S <u>pounds</u> of soybean meal consumed. Answer the following questions:

a) If the price of hogs is $70/<u>cwt</u> (hundredweight), the price of ground corn is $140/<u>ton</u>, and the price of soybean meal is $360/<u>ton</u>, what are each of these prices in $ per pound?

$$\frac{\$70}{1\ cwt} \times \frac{1\ cwt}{1\ lbs} = \$0.70/lb, \quad \frac{\$140}{1\ ton} \times \frac{1\ ton}{2,000\ lbs} = \$0.07/lb, \quad \frac{\$360}{1\ ton} \times \frac{1\ ton}{2,000\ lbs} = \$0.18/lb$$

b) How much ground corn and soybean meal is it economically optimal to feed the feeder pigs? First write out the profit equation using the prices in $ per pound. Second, take the two first derivatives and set them equal to zero. Third, solve these two equations for S and C. Fourth, **check the second order conditions**.

$\pi = 0.7(-585 + 4C + 6S - 0.05C^2 - 0.06S^2 + 0.08CS) - 0.07C - 0.18S$

The partial derivatives of this function with respect to C and S give the following FOC's:
FOC for C: $0.7(4 - 0.10C + 0.08S) - 0.07 = 0.$
FOC for S: $0.7(6 - 0.12S + 0.08C) - 0.18 = 0.$

Several ways exist to solve these two equations for the two variables C and S. I lay out a process below, but it is not the only way. Rearranging the FOC for C gives
    $0.7(4 - 0.10C + 0.08S) = 0.07$
    $4 - 0.10C + 0.08S = 0.07/0.7 = 0.10$
    $3.9 + 0.08S = 0.10C$
    $C = 3.9/0.10 + (0.08/0.10)S$ or $C = 39 + 0.8S$

Substitute this into the FOC for S and solve for S:
    $0.7(6 - 0.12S + 0.08C) - 0.18 = 0$
    $6 - 0.12S + 0.08C = 0.18/0.7 = 0.257$
    $6 - 0.12S + 0.08(39 + 0.8S) = 0.257$
    $6 - 0.12S + 3.12 + 0.064S = 0.257$
    $9.12 - 0.056S = 0.257$
    $9.12 - 0.257 = 8.863 = 0.056S$
    $S = 8.863/0.056 = 158.3$ lbs of soybean meal

Use this S to find C:
    $C = 39 + 0.8S = 39 + 0.8(158.3) = 165.6$ lbs of corn

First I report the second derivatives of π, then check the SOC's.
$d^2\pi/dC^2 = 0.7(-0.10) = -0.07$
$d^2\pi/dS^2 = 0.7(-0.12) = -0.084$
$d^2\pi/dCdS = 0.7(0.08) = +0.056$

SOC's
$d^2\pi/dC^2 = -0.07 < 0$ *satisfied*
$d^2\pi/dS^2 = -0.084 < 0$ *satisfied*
$(d^2\pi/dC^2)(d^2\pi/dS^2) - (d^2\pi/dCdS)^2 = (-0.07)(-0.084) - (0.056)^2 = 0.0027 > 0$
All three SOC's are satisfied for a maximum.

c) When feeding these levels of S and C, what is the final weight of the hogs?

Substitute the optimal C and S found in part a into the weight function:
$W = -585 + 4C + 6S - 0.05C^2 - 0.06S^2 + 0.08CS$
$W = -585 + 4(165.6) + 6(158.3) - 0.05(165.6)^2 - 0.06(158.3)^2 + 0.08(165.6)(158.3)$
$W = 249.7$ *lbs is the final weight*

d) When feeding these levels, what is the net return per feeder pig? Assume net return is pW – $r_cC - r_sS - 110$, where p, r_c and r_s are the prices of hogs, ground corn and soybean meal in $ per pound and $110 is all other costs as $ per pig.

Substitute the results from above into the given returns function:
$\pi = pW - r_cC - r_sS - 120$
$\pi = 0.7(249.7) - 0.07(165.6) - 0.18(158.3) - 110 = \$24.70$ *per pig*