Exports, Innovation and Production Growth:  
A Dynamic Heterogeneous Firm Model with Learning and Entry Costs

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Abstract

We analyze a firm’s joint decision to produce, export, and innovate using a model that incorporates the basic features of self-selection and learning-by-exporting theories of firm-level dynamics. We calibrate the model to 2005-7 Chilean manufacturing plant data and simulate it under different assumptions, finding that neither self-selection nor learning-by-exporting alone can adequately explain the observed cross-sectional relationship between firm-level exports and production, favoring instead a model that requires both mechanisms to work in tandem. Counterfactual policy analysis indicates that the impact of trade liberalization on an industry depends on the maturity of the industry and the nature of the dominant adjustment mechanism governing firm-level export-production dynamics.

JEL Classification: C61; C82; F17; F43; F63; O14; O32.

Keywords: Exports; Production; Innovation; Self-Selection; Learning-by-Exporting; Heterogeneity; Trade Liberalization.

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1 Introduction

It is well documented that firms that export tend to be larger than firms that do not. Two theories have been proposed to explain this observed phenomenon. Self-selection theory, on the one hand, maintains that exporting is a consequence of a firm’s size. Specifically, exporting is profitable. However, to enter the export market, a firm incurs unrecoverable entry costs and typically faces higher fixed-costs of production over time. Entry into the export market is therefore sensible only if the firm is sufficiently large to generate the high volume of exports needed to cover the entry costs and the higher continuing fixed costs. Learning-by-exporting theory, on the other hand, maintains that large firm size is a consequence of entering the export market. Specifically, upon entering the export market, a firm will grow faster as a result of intense yet informative global competition and greater access to state-of-the-art technology.

However, exporting may not be strictly a cause nor strictly a consequence of a firm’s size. That is, exporting and firm growth may mutually reinforcing and jointly determined, in which case neither self-selection theory nor learning-by-exporting theory alone will adequately explain the complex relationship between exports and production.

To study the relationship between exportation and firm size, we develop and analyze a dynamic optimization model of a typical individual firm that allows for both self-selection and learning-by-exporting. Following Melitz [23] and Constantini & Melitz [14], the firm faces unrecoverable export market entry costs, but enjoys faster growth after it becomes an exporter. Each period, the firm must make three key decisions. First, facing an unrecoverable cost of entering the export market, the firm must decide whether to enter, provided that it has not already done so. Second, if the firm enters or has previously entered the export market, it must then decide how much to export and how much to market domestically. Third, the firm, whether an exporter or non-exporter, must decide how much to invest in production-enhancing innovation, the benefits of which may depend on the export status of the firm.

Our model makes a novel contribution to the self-selection and learning-by-exporting trade literatures by allowing export and innovation decisions to be continuous. Specifically, unlike existing models in which innovation and exportation are modelled as discrete, one-time choices, we allow a firm that has entered the export market to sell its product in both the domestic and export markets to degrees that may vary over time and with firm size. Innovation is also viewed as a continuous action whose costs may depend explicitly on whether the firm is or is not an “exporter”.

We extend our single firm model to consider an industry consisting of many firms that are heterogeneous with respect to age, size, and exporter status. Specifically, each period, a fixed percentage of existing firms exit the industry for purely exogenous reasons. Simultaneously, a
fixed number of new, as yet non-exporting firms enter the industry with an initial production drawn from a fixed probability distribution. We calibrate the model using 2005-2007 Chilean manufacturing plant data obtained from the Annual National Industrial Survey of the National Statistics Institute of Chile (INE, Instituto Nacional de Estadisticas Chile).

In our heterogeneous firm industry model, aggregate production, exports, and innovation vary over time, but eventually converge to a steady state. We perform simulations to compare the dynamic adjustments of the industry and the eventual steady-states achieved by the industry under three distinct scenarios: a benchmark scenario that allows for both self-selection and learning-by-exporting, a scenario that allows for self-selection but not learning-by-exporting, and a scenario that allows for learning-by-exporting but not self-selection. We also perform simulations to assess the impacts of trade liberalization on the industry at different stages of industry development, ranging from an infant industry to a mature industry.

We find that neither self-selection nor learning-by-exporting alone can fully explain Chilean manufacturing firm growth and export dynamics from 2005 to 2007. Specifically, without learning-by-exporting, self-selection leads to unrealistically low levels of exports; without self-selection, however, all non-exporters instantly become exporters. Both mechanisms jointly are needed to explain the stylized facts of Chilean manufacturing. We also find that trade liberalization that raises the effective price received by exporters will have a more profound impact on the level of exports in the short- and medium-run if the industry relatively young than when it is mature.

This paper is organized as follows. Section 2 provides a brief review of the existing literature. In Section 3, we develop a model of a representative firm’s joint decisions to export and innovate. In Section 4, we present the benchmark parameter values employed in the study and discuss the data from which the parameter estimates were drawn. In section 5, we derive the representative firm’s optimal export and innovation policies under the benchmark parameters and analyze the dynamic path followed by the optimizing firm. In section 6, we extend the individual firm model to allow for an industry of heterogeneous firms and analyze the industry dynamics. In section 7 we contrast our benchmark model against two polar cases in which either self-selection and learning-by-exporting mechanisms, but not both, are operant. In section 8, we report findings from a counterfactual analysis of the impact of trade liberalization. In Section 9, we discuss our conclusion and offer directions for future research.

2 Literature Review

There is an extensive body of empirical research that supports the hypothesis that investment in technical innovation can reasonably explain the observed positive correlation between firm size and exports. Hallward-Driemeier et al. [18], Baldwin and Gu [6], Bustos [11], and Aw et
al. [4] find that firms’ exports are related to research and development (R&D) investment or adoption of new technology. The key insights drawn from these studies is that innovation and exportation are correlated and that both influence the firm’s growth. Numerous theoretical studies on the role of innovation in exporting decisions have complemented the empirical studies. Atkeson & Burstein [3] and Constantini & Melitz [14] find that trade liberalization increases the rate of return to a firm’s R&D investment, and thus promotes firm growth. In both papers, firms are distinguished only by their size. Wu [29] [30], using industry- and plant-level data, finds that significant learning-by-exporting effects are present only when industries invest substantial amounts in innovation.

Self-selection theory (Melitz [23]; Bernard et al. [8] [9]) also enjoys theoretical support. Empirical tests of learning-by-exporting theory (Marin [22]; Ben-David [7]), however, have produced mixed results, suggesting that the theory may be case-sensitive. For example, studies using data from Indonesia (Amiti & Konings [2]; Blalock et al. [10]), Canada (Baldwin & Gu [8]), UK (Girma et al. [16] [15]) and Chile (Alvarez & López [1]) find strong evidence that exporting firms are more productive. However, Clerides et al. [13], using plant-level data from Mexico, Colombia and Morocco, find no evidence that production costs are affected by entering the export market. Using data from Swedish manufacturing industry, Greenaway et al. [17] find no evidence of pre and post export-market entry difference in firm-level productivity. Using Canadian plant-level data, Lileeva et al. [21] find significant learning effects only when tariffs are cut to promote exports. To resolve the apparent conflict, recent studies (Bustos [12]; Verhoogen [27]; Aw, Roberts & Winston [4]) suggest that learning-by-exporting is possible only if firms make sufficiently high investment in innovation.

Other recent research has compared the mechanisms hypothesized by self-selection and learning-by-exporting theories. Wagner [28] performs a review of the empirical literature, finding strong support for self-selection theory and mixed support for learning-by-exporting theory. He concludes that “exporters are more productive than non-exporters, and the more productive firms self-select into export markets, while exporting does not necessarily improve production”. Imbruno [19] tests for both mechanisms using Italian manufacturing firms, finding that self-selection, rather than learning-by-exporting, is responsible for high production among exporters. Other researchers have combined both mechanisms in a single model to study export-production dynamics. Constantini & Melitz [14] and Aw et al. [5], for example, treat firm-level exporting and innovation as discrete all-or-none decisions.

3 Model of the Firm

The representative firm begins each period either as an established exporter \( i = 1 \) or non-exporter \( i = 0 \), with a predetermined production \( q \geq 0 \). The firm is competitive and thus takes
the domestic market price $P_0$ and the export market price $P_1$ as given (hereafter, $P = (P_0, P_1)$ indicates the joint price vector). The firm must then decide whether to assume exporter status $j = 1$ or not $j = 0$ for the current period, and, if it assumes exporter status, must decide what proportion $y$ of its production $q$ to export, marketing the remainder domestically.

The firm must also decide how much to invest in innovation that enhances the firm’s productive capacity the following period. Specifically, the firm’s production in the following period

$$q' = (1 - \gamma)q + x$$

is a function of its current production $q$, the amount it invests in innovation $x \geq 0$, and the per-period rate of production depreciation $\gamma \in (0, 1)$.

Each period the firm faces a fixed cost $\kappa_1$ if it is an exporter ($j = 1$) and a fixed cost $\kappa_0$ if it is not an exporter ($j = 0$), where $\kappa_1 > \kappa_0 > 0$. A non-exporter that becomes an exporter incurs a fixed unrecoverable investment $K > 0$. To economize on notation, we let $\eta_{ij}$ denote the sum of fixed production and export market entry costs associated with the firm choosing exporter status $j$ this period, given it begins the period in exporter status $i$. Thus the export-status switching cost is:

$$\eta_{ij} = \begin{cases} 
\kappa_1 + K & \text{if } i = 0, j = 1 \\
\kappa_j & \text{if otherwise.} 
\end{cases}$$

The firm also incurs a quadratic variable production and marketing cost:

$$D(q, y) = d_0(q(1 - y))^2 + d_1(qy)^2; \quad (2)$$

and incurs a variable innovation cost:

$$C_j(x) = c_j x^\beta \quad (3)$$

with $\beta > 0$. We assume that marketing in the export market is more expensive than marketing in the domestic market ($d_1 > d_0 > 0$) and, to capture learning-by-exporting, that an exporter faces lower innovation costs than a non-exporter ($c_0 > c_1 > 0$).

The firm maximizes the present value of current and expected future profits over an infinite horizon, given that it faces a probability $\mu$ of going bankrupt and ceasing to exist each period. The firm’s dynamic decision problem is characterized by a Bellman equation whose value function specifies the maximum present value of current and expected future profits $V_i(q)$ attainable by the firm, given its export status $i$ and production $q$ at the beginning of the period:

$$V_i(q) = \max_{j=0,1 \atop 0 \leq y \leq \frac{q}{\gamma} \atop x \geq 0} \{P_0 q(1 - y) + P_1 q y - C_j(x) - D(q, y) - \eta_{ij} + \delta(1 - \mu)V_j((1 - \gamma)q + x)\}. \quad (4)$$

Here, $\delta \in (0, 1)$ is the firm’s per-period discount rate.
The industry is assumed to consist of many firms that behave as the representative firm, but which differ with respect to age and initial size. We assume a fixed proportion $\mu$ of firms exit the industry randomly in any period but are replaced by a fixed number of new firms with production drawn randomly from a known distribution. Specifically, we assume that a nascent firm’s initial production is drawn randomly from a distribution $F$. Idiosyncratic variation in initial production and time of death produces an industry with firms that are heterogenous with respect to age, size, and export status; the total number of firms in the industry increases until the number of firms exiting equals the number of firms entering.

4 Model Parameterization

We parameterize our model by calibrating it to match 2005-7 Chilean manufacturing plant data compiled in the National Industrial Survey conducted by the Chilean National Statistics Institute (INE, Instituto Nacional de Estadisticas). The annual INE survey includes plants that began operating during that year and excludes those that closed during the year for any reason. In the survey, production, exports, overseas business connections, and other manufacturing information are collected at the plant level. Although firms may own more than one plant, more than 90% of Chilean manufacturing firms have exactly one plant (Pavcnik [26]).

Table 1 summarizes the main characteristics of the plants sampled in the annual INE survey. As seen in Table 1, among the 6,614 plants sampled, 1,624 (25%) are exporters, while 4,990 (75%) are non-exporters. “Ownership Structure” refers to the proportion of domestic and foreign ownership interests in the plants. Clearly, foreign ownership interest tends to be greater among exporters than non-exporters. Specifically, 8.7 percent of exporters have some foreign ownership, whereas only 2.4 percent of non-exporters have some foreign ownership; also, 10.3 percent of exporters are fully foreign owned, whereas only 0.7 percent of non-exporters are fully foreign owned.

Table 1 shows the distribution of plant size, for both exporters and non-exporters, by number of workers: “small”, 10-49 workers; “medium”, 50-149 workers; and “large”, over 150 workers. Clearly, exporters tend to be larger than non-exporters. Specifically, the percentage of exporters that are large (3.7%) exceeds the percentage of non-exporters that are large (0.4%), and the percentage of non-exporters that are small (87.7%) exceeds the percentage of exporters that are small (41.2%).

Table 1 gives the distribution of the productivity of both exporters and non-exporters. The distribution of the productivity of the exporters is much more right skewed than that of the non-exporters. Only very few exporters (3.5%) are featured with productivity lower than 0.5, while 25.1% of the non-exporters belong to the same low-productivity group. Meanwhile, nearly 2.0% of the exporters have productivity higher than 3, while only very few (0.5%) of
the non-exporters have such high productivity.

Table 1 also gives the number of exporting and non-exporting plants in the INE survey that undertook some notable innovation in its capital assets, including buildings, vehicles, machines, and land. As can be seen in Table 1, most plants report having undertaken no significant capital innovation in the year they were surveyed. However, the percentage of exporters that report at least one capital innovation (11.6%) is nearly twice the percentage of non-exporters that report at least one capital innovation (6.6%).


<table>
<thead>
<tr>
<th></th>
<th>Exporters</th>
<th>Non-Exporters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Percentage</td>
</tr>
<tr>
<td>NUMBER OF PLANTS</td>
<td>1,624</td>
<td>100.0</td>
</tr>
<tr>
<td>OWNERSHIP STRUCTURE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Foreign Ownership</td>
<td>1,321</td>
<td>81.3</td>
</tr>
<tr>
<td>Mixed Ownership, Mostly Domestic</td>
<td>49</td>
<td>3.0</td>
</tr>
<tr>
<td>Mixed Ownership, Mostly Foreign</td>
<td>86</td>
<td>5.3</td>
</tr>
<tr>
<td>Fully Foreign Owned</td>
<td>168</td>
<td>10.3</td>
</tr>
<tr>
<td>PLANT SIZE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small (10-49 Workers)</td>
<td>669</td>
<td>41.2</td>
</tr>
<tr>
<td>Medium (50-149 Workers)</td>
<td>895</td>
<td>55.1</td>
</tr>
<tr>
<td>Large (≥ 150 Workers)</td>
<td>60</td>
<td>3.7</td>
</tr>
<tr>
<td>PRODUCTIVITY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∈ [0 0.5)</td>
<td>56</td>
<td>3.45</td>
</tr>
<tr>
<td>∈ [0.5 1)</td>
<td>391</td>
<td>24.08</td>
</tr>
<tr>
<td>∈ [1 3)</td>
<td>1,145</td>
<td>70.50</td>
</tr>
<tr>
<td>≥ 3</td>
<td>32</td>
<td>1.97</td>
</tr>
<tr>
<td>CAPITAL INNOVATIONS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Innovations in Buildings</td>
<td>122</td>
<td>7.5</td>
</tr>
<tr>
<td>Innovations in Vehicles</td>
<td>15</td>
<td>0.9</td>
</tr>
<tr>
<td>Innovations in Machines</td>
<td>101</td>
<td>6.2</td>
</tr>
<tr>
<td>Innovations in Land</td>
<td>7</td>
<td>0.4</td>
</tr>
<tr>
<td>No Innovation</td>
<td>1,436</td>
<td>88.4</td>
</tr>
</tbody>
</table>

Table 2 reports the parameter values used for our benchmark model simulations. In our parameter specification, all prices and costs are expressed relative to the domestic price $P_0$, which is normalized to 1. Following Constantini & Melitz [14] we set the discount factor $\delta$ to 0.95. The remaining parameters can be divided into three groups. First, parameters estimated directly from the INE Survey data, including the fixed cost of exporters $\kappa_0$ and non-exporters $\kappa_1$, as well as the product depreciation rate $\gamma$. Second, parameters that are chosen through calibration, that is, so that simulated values of prescribed endogenous variables match empirically observed values; these parameters include the bankruptcy probability $\mu$ and export market entry cost $K$. Third, parameters whose values are estimated econometrically, including the export price $P_1$, the innovation cost function parameters $c_0$ and $c_1$, the production cost function parameters $d_0$ and $d_1$, the innovation rate $\beta$; the column “Range” gives the 95% confidence intervals for the point estimates.

Table 2: Simulation Model Baseline Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Benchmark Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Price</td>
<td>$P_0$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Export Price</td>
<td>$P_1$</td>
<td>2.0</td>
<td>1.75 ~ 2.17</td>
</tr>
<tr>
<td>Export Start-Up Cost</td>
<td>$K$</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost - Non-Exporter</td>
<td>$\kappa_0$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost - Exporter</td>
<td>$\kappa_1$</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Innovation Cost Parameter - Non-Exporter</td>
<td>$c_0$</td>
<td>0.9</td>
<td>0.98 ~ 1.00</td>
</tr>
<tr>
<td>Innovation Cost Parameter - Exporter</td>
<td>$c_1$</td>
<td>0.23</td>
<td>0.08 ~ 0.37</td>
</tr>
<tr>
<td>Production Cost Parameter - Domestic Products</td>
<td>$d_0$</td>
<td>0.2</td>
<td>0.20 ~ 0.24</td>
</tr>
<tr>
<td>Production Cost Parameter - Export Products</td>
<td>$d_1$</td>
<td>0.85</td>
<td>0.33 ~ 1.31</td>
</tr>
<tr>
<td>Innovation Cost Rate</td>
<td>$\beta$</td>
<td>10</td>
<td>9.12 ~ 12.19</td>
</tr>
<tr>
<td>Bankruptcy Probability</td>
<td>$\mu$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Production Depreciation Rate</td>
<td>$\gamma$</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Discount Rate</td>
<td>$\delta$</td>
<td>0.95</td>
<td></td>
</tr>
</tbody>
</table>

We use the average domestic sales tax rate to estimate the fixed cost $\kappa_0$ faced by non-exporters; to this we add the average tax rate on exports to estimate the fixed cost $\kappa_1$ faced by exporters. The average capital depreciation rate reported in the INE dataset is used to estimate the productivity depreciation rate $\gamma$.

To calibrate export market entry costs $K$ and bankruptcy probability $\mu$, we use two targets from the data: the percentage of aggregate exported products and the percentage of plants that are exporters. Among the Chilean manufacturing plants surveyed, the export supply
percentage is 16% and 25% of plants produce output for export.

We use the equilibrium Euler equations with the DYNARE package in Matlab to estimate the export price $P_1$, the innovation cost parameters $c_0$ and $c_1$, the production cost parameters $d_0$ and $d_1$, as well as the innovation cost rate $\beta$. All these parameters appear in the equilibrium equations. Given with the data and the equations, DYNARE systematically gives econometric estimation of these parameters. We use the empirical distribution of the productions of newborn firms between 2005 and 2007 to indicate $F$. The firm-level productions are reported in Wu [30], who used an extended Olley-Pakes method to complete the estimation from the INE data set (Olley & Pakes [25]).

5 Firm Dynamics

We begin with an analysis of the representative firm’s optimal export and innovation policies and the path taken over time by an individual firm that follows these policies.

The firm’s Bellman equation (4) is solved using the method of collocation (Judd [20]; Miranda and Fackler [24]). The collocation method calls for the value functions $V_i(q)$ to be approximated using a linear combination of $n$ known basis functions $\phi_o$:

$$V_i(q) \approx \sum_{o=1}^{n} \alpha_{io} \phi_o(q)$$

(5)

The $2n$ unknown coefficients $\alpha_{io}$ are then fixed by requiring the value function approximate to satisfy the Bellman equation (4), not at all production levels $q$, but rather at $n$ judiciously chosen collocation nodes $q_o$. The collocation method replaces the Bellman functional equations with a set of $2n$ nonlinear equations with $2n$ unknowns that are solved using Newton’s method. The collocation method can generate highly accurate approximate solutions to the Bellman equation, provided the basis functions and collocation nodes are chosen judiciously and their number $n$ is set adequately high. For this paper, we chose orthogonal polynomial basis functions and $n = 150$ nodes to compute the approximate solutions for the Bellman equation. The solution was computed using the CompEcon2012 Toolbox routine DPSOLVE (Miranda and Fackler [24]).

Figure 1 illustrates the conditional value functions for a current non-exporter under the benchmark parameterization. The horizontal axis represents the firm’s level of production $q$ and the vertical axis represents the expected present value of profits of a non-exporting firm over the firm’s indefinite time horizon. For a current non-exporter, the red curve represents the value associated with becoming an exporter and the blue curve represents the value of remaining a non-exporter. The two curves intersect each other at the critical firm size $q^* = 2.71$ at which a non-exporter becomes an exporter.
Figure 1: Conditional Value Functions for Non-Exporter

Figure 2 illustrates the representative firm’s optimal export policy (left panel) and optimal innovation policy (right panel). On both panels the horizontal axis represents the firm’s size $q$. As seen in the left panel of Figure 2, if $q < q^*$, a non-exporting firm remains a non-exporter; if $q > q^*$, a non-exporting firm becomes an exporter and, over this range of production, the proportion of output exported declines with production due to rising production and marketing costs.

Figure 2: Optimal Export and Innovation Policies for Typical Firm

As seen in the right panel of Figure 2, the firm’s optimal innovation policy makes a discrete jump at the critical production level $q^*$ at which the firm becomes an exporter. However, once the firm becomes an exporter, innovation decreases with production, eventually falling
to zero. At sufficiently high levels of production, the firm sets investment in innovation below the minimal replacement level in order to allow production to fall so as to reduce its marketing costs; if the firm is the firm is sufficiently large, it stops innovating altogether.

As seen in the right panel of Figure 2, optimal innovation initially declines with production, but begins to rise in an irregular manner as production approaches the critical level $q^*$ at which the firm becomes an exporter. This is seen more clearly in Figure 3, which magnifies the firm’s optimal innovation policy below the export transition point. The five kinked points in Figure 4 correspond to critical levels of output at which the number of periods remaining to becoming an exporter change. If $q < q_5^* = 1.04$, the firm is 5 periods from becoming an exporter; if $q_5^* = 1.04 \leq q < q_4^* = 1.58$, the firm is 4 periods from becoming an exporter; and so on. Although innovation increases at each critical $q_m^*$, it declines between the $q_m^*$.

![Figure 3: Optimal Innovation of a Non-Exporter](image)

The individual firm evolves over time, each period facing an exogenous probability of dying $\mu$. It therefore dies in finite time with probability 1. Regardless of its size at birth, however, the typical firm, if it lives lives long enough, will become an exporter and its production $q$, innovation $x$, and proportion of output exported $y$ converging to hypothetical steady-state values of 3.57, 0.71, and 0.32, respectively.

Consider a firm that starts as a non-exporter with initial production $q = 0.1$ and lives indefinitely. As seen in Figure 4, the firm innovates from the outset and its production grows. In period 8, the firm’s production reaches the critical level $q^*$, at which point it becomes an “exporter”. From that point forward, the firm’s production continues to grow, but at a declining rate, eventually approaching the steady-state of 3.57.

The dynamics of exports and innovation for a typical firm are illustrated in Figure 5. As seen on the left panel, the firm starting with $q = 0.1$ does not export until the 8th period;
once this happens, the proportion of production exported immediately jumps to its highest level. However, as exporter continues to grow, the proportion of production exported declines gradually because production cost increase.

As seen on the right panel of Figure 5, a nascent non-exporting firm innovates at first, but a declining rate due to rising costs of innovation. However, as a non-exporting firm’s production approaches the critical level $q^*$ at which the firm becomes an exporter, the firm’s production begins to rise as the firm anticipates becoming an exporter and being able to take advantage of the attendant lower innovation cost. Innovation reaches its maximum in the period when the firm becomes an exporter. Once the firm becomes an exporter, its innovation begins to decline, again due to rising marginal cost of innovation, eventually converging to its steady-state value, the level of innovation needed to cover depreciation in steady-state.
6 Industry Dynamics

We now examine the dynamics of the entire industry, allowing for exogenous entry and exit that generates heterogeneity in size, exporter status, and innovation across firms. To illustrate industry dynamics, we simulate the industry from its hypothetical birth, at which time a fixed number of firms enter the market in accordance with the calibrated entrant size distribution. Each period, an proportion $\mu$ of existing firms exit the industry, and a fixed number of new firms enter with sizes randomly drawn from the calibrated entrant size distribution function. New firms are non-exporters. However, with time, firms grow, and those that survive sufficiently long eventually enter the export market.

Figure 6(a) shows total industry output during this period. As seen in Figure 6, total industry production rises rapidly at first, but eventually levels off. Figure 6(b) shows the percentage of firms that are exporters and the percentage of production that is exported from the birth of the industry in period 0 through period 35. Also, all firms begin as non-exporters and the industry as a whole does not begin to export until period 6, after which the percentage of exporters and production exported grow rapidly and eventually level off. With time, the percentage of firms that are exporters, the percentage of production that is exported, and total industry output converge to steady-state values of 16%, 25%, and 1.93, respectively.

Figure 6: Aggregate Industry Dynamics over Time

Figure 7(a) shows the steady-state cumulative distribution function of firm sizes. The critical firm size $q^* = 2.71$ at which a non-exporter becomes an exporter is clearly indicated. As can be seen in the figure, 25% of all firms in steady-state are exporters. The distribution past the critical firm size is very much right-skewed.
Figure 7: Steady-State Industry Distribution

(b) shows the steady-state cumulative distribution of export ratio. 75% of the firms have zero export ratio, as they are non-exporters. The exporters’ proportion of products exported ranges between 0.33 to 0.37, and the distribution among them is very left-skewed.

We now perform sensitivity tests in order to analyze how the industry steady-state depends on key model parameters. In particular, we examine how the quantity of total industry production that is exported and marketed domestically in steady-state vary with the export price, the non-exporter innovation costs, exporter production costs, and domestic production costs.

Figure 8: Steady-State Industry Export and Domestic Sales as Functions of the Export Price
6.1 Export Price

Figure 8 gives the steady-state industry export and domestic sales as functions of the export price $P_1$ (recall that the domestic price $P_0$ is normalized to 1). As seen in figure 8, if the export price is less than 1.95, firms will never choose to export, even if the product sells at a higher price in the export market, because the benefits of higher profitability and lower innovation costs are insufficient to cover the fixed costs of entering the export market. If the export price exceeds 1.95, firms that survive sufficiently long eventually enter the export market. A higher export price implies greater exports and lesser domestically marketed production.

6.2 Innovation Cost

Figure 9(a) gives the steady-state industry export and domestic sales as functions of the exporter's innovation cost $c_1$. As seen in the figure, higher non-exporter innovation costs have no significant influence on domestic sales, but will lower the export sales. As the exporter innovation cost rises, it takes longer for non-exporters to jump; there will be less exporters and export sales.

Figure 9(b) gives the steady-state industry export and domestic sales as functions of the non-exporter's innovation cost $c_0$. As we can see, higher non-exporter innovation costs lower production and both export and domestic sales. As the non-exporter innovation cost rises, it also takes longer for non-exporters to enter the export market, slowing down their growth.

Figure 9: Steady-State Industry Export and Domestic Sales as Functions of the Innovation Cost
6.3 Production Cost

Figure 10(a) gives the steady-state industry export and domestic sales as functions of the exporter production cost $d_1$. As seen in figure 10(a), domestic sales increase and export sales decrease as the exporter production cost increases. Notice that the domestic and export supply curves intersect at $d_1 = 0.47$. When $d_1 < 0.47$ aggregate export sales dominate the domestic sales, otherwise the aggregate domestic sales exceed the export sales. Exporting becomes prohibitively expensive once the export production costs exceeds 0.89; increases in the cost beyond this point no longer affect the steady-state equilibrium.

Figure 10(b) gives the steady-state industry export and domestic sales as functions of the domestic production cost $d_0$. As depicted in the figure, domestic production falls as domestic production costs increase. However, as long as domestic production costs remain below 0.145, holding export production costs constant, domestic production will remain more profitable than export production and no firms will export. Once domestic production costs rise above this level, however, it becomes profitable for firms to export once they reach the required size, and at the industry level, domestic marketings fall and export marketings rise with the cost of domestic production.

Figure 10: Steady-State Industry Export and Domestic Sales as Functions of the Production Cost

6.4 Fixed Cost

Figure 11(a) gives the steady-state industry export and domestic sales as functions of $\kappa_1$, the fixed cost a firm needs to pay once it enters the export market. Assume the foreign tariff rises, so does the cost. As $\kappa_1$ increases, it costs more to stay in the exporting market. If $\kappa_1 < 1.54$, aggregate export supply dominates the domestic supply; otherwise aggregate domestic
supply exceeds the export supply. At $\kappa_1 = 1.54$, export supply and domestic supply are equal and each of them takes 50% of the aggregate production in the industry.

When $\kappa_1 < 2.06$ a non-exporter is able to jump eventually. As $\kappa_1$ rises the aggregate export supply decreases, while the domestic supply increases. Once $\kappa_1 > 2.06$, the fixed exporting cost will be so high that a non-exporter can never make the jump, and there will be no exporters on the market in the steady-state. The domestic supply keeps constant, without any export supply in the industry.

Figure 11: Steady-State Industry Export and Domestic Sales as Functions of the Fixed Cost

Figure 11(b) gives the steady-state industry export and domestic sales as functions of the non-exporter’s fixed cost $\kappa_0$. As $\kappa_0$ rises, it costs more to merely stay in the domestic market. If $\kappa_0 < 0.92$, a non-exporter can never enter the export market because of the high fixed cost; there are no exporters on the market and the domestic sales keep constant. Once $\kappa_0 > 0.92$ non-exporters can jump eventually. As $\kappa_0$ rises, the aggregate export supply increases while the domestic supply decreases. At $\kappa_0 = 1.43$ the export supply and domestic supply are equal.

6.5 Exporting Start-Up Cost $K$

Figure 12 gives the steady-state industry export and domestic sales as functions of the exporting start-up cost $K$. If $K < 1.38$, non-exporters can make the jump if they can survive long enough in the market. As $K$ increases, the domestic supply will increase and the export supply decrease. Once $K > 1.38$ no exporters will exist anymore; the production will be purely domestic. No matter how $K$ varies, the domestic production keeps being constant.
6.6 Bankruptcy Probability \( \mu \)

Figure 13 gives the steady-state industry export and domestic sales as functions of \( \mu \), the probability of a firm being forced to exit the market. Each period a firm faces a chance of \( \mu \) to stop production and discontinue its existence on the market. As \( \mu \) goes higher, it is more likely for a firm to exit the market. Recall that under the benchmark assumption all the new firms are all born as non-exporters. Thus both domestic and export sales will decrease.
7 Self-Selection vs. Learning-by-Exporting

How do self-selection and learning-by-exporting affect firm and industry dynamics? To unravel the effects of self-selection and learning-by-exporting, we simulate the model under two extreme polar cases: one that allows for self-selection but not for learning-by-exporting, and one that allows for learning-by-exporting but not for self-selection. By construction, the export supply percentage and exporter percentage observed in our INE dataset are replicated in the benchmark model. As we will see, if remove either of the two mechanism, it is no longer possible to meet both targets.

Figure 14: Percentage of Exporting Firms and Percentage of Production Exported under Pure Self-Selection and Pure Learning-by-Exporting Models

7.1 Pure Self-Selection Model

We eliminate learning-by-exporting from our model by setting the innovation cost for exporters $c_1$ to be as high as the innovation cost for non-exporters $c_0$. Under this assumption, the firm’s export market participation has no influence on its innovation costs. The only cost for a firm entering the export market is the cost of entry associated with self-selection theory as reflected in the fixed export cost and export start-up cost. Keeping other parameters at their benchmark values, we again simulate an individual firm that begins as a non-exporter with production level $q = 0.1$. In this model, the firm remains a non-exporter for 8 periods before entering the export market. As for the industry as a whole, relative exporting supply falls by 30% and the exporter percentage falls by 38%.

Next we attempt to recalculate the model by varying the export start-up cost $K$ and
bankruptcy probability $\mu$, respectively. We find, however, that it is not possible to replicate both of our targets simultaneously. The horizontal axis of Figure 14(a) represents the recalibrated value of $K/\mu$. The red curve gives the percent of firms that are exporters and the blue curve gives the percent of production that is exported. At point A, the exporter percentage is replicated. Because of the absence of the learning effect, exports are discouraged; even with if percentage of firms of firms that export remain unchanged. As a result, the export supply percentage is only 13%, which is 19% lower than in the empirical data. If we attempt to match the export supply ratio to the data, as in point B where the export supply percentage is replicated, then the exporter percentage will be 31%, which is 24% higher than in the data.

Under a pure self-selection mechanism, the domestic and export markets are completely separated. Hereby I take case B, the one with only exporter percentage being matched for example. At the firm-level steady-state, the export ratio rises by 3%; innovation decreases by 4%; the production also by 28%. At the industry-level, when the exporter percentage is replicated, the export supply percentage decreases by 19%.

Thus, without learning-by-exporting mechanism, the positive correlation between exports and production can only be partially explained; the two empirical targets cannot be replicated simultaneously.

Figure 15: Growth Paths of a Firm Starting with $q = 0.1$ under a Recalibrated Pure Self-Selection Mechanism

Figure 15 depicts the simulated growth path of a firm beginning from $q = 0.1$ after recalibration, when only the exporter percentage is replicated from the data. It still takes 8
periods for this firm to become an exporter, the same as in the benchmark model. Notice in this case, innovation investment peaks in the 7th period, one period before switching. Afterwards innovation declines gradually because of the increasing marginal cost. Production increases smoothly at a decreasing rate, experiencing a bump in the 7th period because of the innovation peak. Apart from these differences, all the other findings are similar to the benchmark model.

### 7.2 Pure Learning-by-Exporting Model

We eliminate self-selection from our model by setting the export start-up cost $K$ to 0. In this case, high export prices and learning-by-exporting provide strong motivation for firms to export. More specifically, non-exporting firms have the incentive to immediately become exporters because the change in status does not impose additional costs and innovation cost fall. In the absence of start-up costs, firm grow faster. The export supply percentage in steady-state will be 39% and the aggregate exporter percentage will be 85%, both of which are unrealistically high.

Next we attempt to recalibrate the model by systematically varying the export price $P_1$ and the bankruptcy probability $\mu$. We find that whenever the export price exceeds the domestic price, non-exporters will always chose to become an exporter immediately. Even when the innovation cost are the same for exporters and non-exporters, in which case there is no learning-by-exporting effect, exporting is desirable due to the higher profit. To replicate our empirical targets, we need to allow for a lower export price. Specifically, $P_1$ should be 92% of $P_0$. At the same time, the bankruptcy probability must increase by 50%.

The horizontal axis of Figure 14(b) represents the recalibrated value of $P_1/\mu$ from the benchmark level. The red curve gives the percent of firms that are exporters and the blue curve gives the percent of production that is exported. Regardless of how we vary the parameters, the simulated exporter percentage exceeds 65%, which is unrealistically high. The exporter percentage in the data is only 25%. Thus, we conclude that the pure learning-by-exporting can only successfully replicate the relative export supply, given by point C. Without a self-selection mechanism and a higher export price, the positive correlation between exports and production can be explained only with an unrealistic percentage of firms exporting.

Figure 16 depicts the simulated growth path of a firm beginning from $q = 0.1$ after recalibration. It now takes 2 periods for a non-exporter to jump. Even with a lower export price, being an exporter is still attractive because exporting reduces the cost of innovation. Innovation investment is greatest in the very first period, then gradually decreases due to the increasing marginal cost. The steady-state levels of innovation investment and production both decrease by 15%, and the export ratio decreases by 53%. At the industry level, the export supply percentage can be replicated, but the exporter percentage is as much as over
three times greater in the data.

Table 3 summarizes the results of our two polar case simulations. Column (i) shows the results of the benchmark model which incorporates both self-selection and learning effects. Columns (ii) and (iii) show the two scenarios of results under the pure self-selection model with either stylized fact is replicated. The firm-level steady states are the same; however the industry-level exporter percentage and the export supply percentage cannot be replicated simultaneously. Column (iv) shows the results under the pure learning-by-exporting mechanism with the inconvincibly high exporter percentage.

Generally speaking, self-selection tends to work at the industry-level, allowing more productive firms into the export market. Because of the entry costs, self-selection requires that a non-exporter grow before it starts to export. However, learning-by-exporting works more at the the firm-level; once a firm becomes an exporter, it benefits from a lower innovation cost.

Without one or the other mechanism, the model fails to replicate the data adequately. Pure self-selection can only replicate one stylized fact at one time; pure learning-by-exporting can only replicate the export supply percentage but with a much higher exporter percentage. Only with both mechanisms can the model adequately explain both stylized facts.
Table 3: Steady-State Values of Selected Variables under the Pure Self-Selection, Pure Learning-by-Exporting, and Benchmark Models

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<sup>1</sup>: It indicates how many periods for a firm starting with q = 0.1 to switch from a non-exporter to an exporter.

<sup>2</sup>: Reports the steady-state simulation results on the firms.

<sup>3</sup>: Reports the steady-state simulation results of the aggregate industry.

8 Effects of Trade Liberalization

In this section we examine the effects of trade liberalization that increases the effective price received for exports, say through a reduction of export tariffs at home or a reduction of import tariffs abroad. To be specific, assume that the export price $P_1$ increases from 2 to 2.2. This change occurs gradually over 5 periods, with $P_1$ each period increasing by 0.04. We will analyze the changes of firm-level dynamics under different scenarios in details. We then make comparison between the counterfactual and the previous benchmark results to determine the influence of different liberalizations.

The first counterfactual case is a liberalization after the aggregate industry has reached its steady state. In the benchmark model, at the 40th period the industry will be stabilized. Now assume that an industry is mature enough to have achieved steady-state before liberalization, then it experiences by a decrease of the transportation cost. As a result, the real export price is raised by 10%. Because of this change, firms will make different decisions regarding their innovation and export ratio. This will lead to changes in aggregate supplies and exporter percentage from time to time.

The second counterfactual case assumes a trade liberalization occurs before the industry
reaches steady state; $P_t$ rises when the industry is still at its initial stage of growth. To be specific, the liberalization occurs to a young industry at its 5th period of growth, when there was no exporters yet. We then look into how the dynamics of the industry differs across scenarios under different liberalization assumptions.

Figure 17: Comparison of Aggregate Export Supply Percentage Dynamics After Trade Liberalization

The horizontal axis in Figure 17 indicates the period after the liberalization finishes. Figure 17(a) depicts the dynamics of export supply percentage. If the liberalization happens after the industry is already stabilized, as shown by the blue line, the export supply percentage will both increase slowly period by period. To be specific, after 30 periods the export supply percentage will grow by 43%. If the liberalization occurs to a young industry which is far from its steady state, as the red line shows, the export supply percentage will immediately grow rapidly. However, it keeps below the mature-industry-case level; moreover, it is lower than the contemporaneous benchmark level for 10 periods. If this gradual liberalization occurs to a young industry, it will cause a lower aggregate export supply.

Figure 17(b) depicts the dynamics of the exporter percentage. Still, the horizontal axis indicated the period after the liberalization. For a mature industry as shown by the blue line, the exporter percentage grows gradually period by period, and after 30 periods the exporter percentage will grow by 27%. However for a young industry, the exporter percentage will be much lower, even lower than contemporaneous benchmark level for 20 periods. If the liberalization lasts for 10 periods, the exporter percentage stays below the benchmark result even after the 40th period. Therefore surprisingly, if the liberalization happens to an industry that is so young that no exporters have appeared yet, it will reduce the industry’s exporter percentage.
To sum up, trade liberalization’s influence depends on when it happens. If the liberalization happens to mature industry which has already reached its steady state, the export intensity of the industry will gradually increase. But if the liberalization happens to a young industry, the export supply percentage and the exporter percentage will both be at lower levels for a long while.

9 Conclusion

In this paper, we have developed a dynamic model of a firm’s joint decisions to export and innovate, allowing both decisions to affect the firm’s production in accordance with self-selection and learning-by-exporting theories of firm-level export and production dynamics. The model is extended to allow for an industry containing many firms that are heterogeneous with respect to age, export status, and production.

We find that both self-selection and learning-by-exporting critically influence the dynamic evolution of a firm, an thus the dynamic evolution of an industry. On the one hand, models that allow only for self-selection can replicate the percentage of firms that export, but predict a much lower volume of exports than is observed. On the other hand, models that allow only for learning-by-exporting can replicate the relative volume of exports, but predict unrealistically high percentage of firms involved in exporting. To adequately capture the stylized facts of Chilean manufacturing from 2005 to 2007, both mechanisms must be posited.

In a counterfactual policy analysis, we find that the effects of trade liberalization depend critically on the age of the industry. On the one hand, trade liberalization for a mature industry encourages exports. However if the trade liberalization happens to a young industry, the export intensity of the industry will be reduced.

There remain many potentially interesting extensions of the work presented in this paper. For example, the empirical studies on the model’s predictions regarding the impacts of trade liberalization can be carried out. Another theoretical extension would be to allow firms to shut down temporarily, rather than shutting down forever. Finally, one could extend the model by explicitly incorporating a production function and factor pricing. This would allow us to study the influence of wages on firm- and industry-level dynamics and the impacts of labor immigration and wage policies.
References


Appendix

A Sensitivity Tests

A.1 Production Depreciation Rate $\gamma$

![Figure 18: Steady-State Industry Export and Domestic Sales as Functions of the Production Depreciation Rate $\gamma$](image)

Figure 18 shows the sensitivity test based on $\gamma$, the depreciation rate of production between periods. As $\gamma$ increases, consequently $q$ will grow slower. Specifically, as $\gamma$ stays below 0.25 and increases, the periods it takes for a non-exporter to jump to become an exporter will increase since it slows down firms’ production growth. As a result, there are less aggregate export supply.

Once $\gamma$ exceeds 0.25, the production depreciation is so high that non-exporters can never make the jump. There will be no exporters in the industry. No export supply will exist; the aggregate production supply, which is purely domestic, decreases as $\gamma$ increases.

A.2 Innovation Cost Rate $\beta$

Let’s look at how $\beta$ affects the steady-state results of the model. $\beta$ is a firm’s innovation cost rate. The higher $\beta$ becomes, the less it costs a firm, no matter a non-exporter or an exporter, to invest in the same amount of innovation. In Figure 19, as we increase $\beta$ from 5.5 to 12, it costs less to innovate. The periods it takes for a new-born firm to jump will decrease. Non-exporters’ jumping decisions will be accelerated. In the aggregate industry, both domestic and export supply will increase.
Figure 19: Steady-State Industry Export and Domestic Sales as Functions of the Innovation Cost Rate $\beta$

![Graph showing the relationship between steady-state industry export and domestic sales as functions of the innovation cost rate $\beta$.]

Figure 20: Conditional Value Functions for Exporter

![Graph showing conditional value functions for exporter.]

B Figures
Figure 21: Growth Paths of a Firm Starting with $q = 0.1$ under a Pure Self-Selection Mechanism

Figure 22: Growth Paths of a Firm Starting with $q = 0.1$ under a Pure Learning-by-Exporting Mechanism