

# Economic Development and Industrial Dynamics of Technological Change

Nima Veisoh\*  
Georgetown University

March 29, 2013

## Abstract

This paper makes a contribution toward identifying market failures impeding innovation and mechanisms through which a Socially Optimal level of technological investment can be achieved. The novel framework illustrates interaction between agents driving the Solow Residual. We lay the foundation for a class of models, starting most simply with a Normal Form Coordination game, which describes the interaction between the Firm, the originator of R&D, and Human Capital, without whom the Firm cannot perform research. Other agents include, the Investor, who funds Firms, and the Government, which has the choice of subsidizing innovation at the Firm level.

The contribution is three-fold. First, we show how increasing complexity and changes in access to private capital markets raised the de facto cost of performing research, leading many Firms to abandon in-house innovation. Second, we characterize an equilibrium as a function that provides the required subsidy rate of the Government to innovating Firms, in order to sustain an equilibrium where not only do Firms and Human Capital both decide to dedicate themselves toward innovation, but satisfies Investors needs for a return on capital. Third, that the equilibrium function describing the subsidy required of the Government is non-linearly increasing in time. Additionally, given the endogenous feedback of technological progress on economic growth, i.e. more technological investment leads to more government revenues, the equilibrium can actually be sustained over an infinite time horizon.

\*This working paper is presented at the 10th Midwest International Economic Development Conference, in Madison, WI. I gratefully acknowledge the help and insights of John Rust, and the support of the Breakthrough Institute and Georgetown University. Please forward comments, critiques and inquiries to [nimav@alum.mit.edu](mailto:nimav@alum.mit.edu), as feedback is appreciated. All errors are my own.

# 1 Introduction

Private market funding alone is not sufficient to provide a sustainable, long-run rate of technological development. Empirically, we observe that as technology progresses (naturally increasing in complexity over time), the marginal cost of innovation increases as well. [1] Thus, an endogenous level of outside-public-government funding may be required to subsidize private agents in technological development, such that Firms can satisfy the zero-profit constraint. The risk-return profile required by Investors of Firms in the private competitive market deters Firms from investing in technological research of increasing complexity, which may have a smaller probability weighted payoff, as there is a degree of uncertainty that goes into basic technological research. The stochastic nature of innovation means that Firms do not know when basic research will result in profits. Lastly, as the risk-return profile of a research Firm falls below the market's Risk-Free rate, we expect that such basic research Firms will die off.

We model the comparative statics through a Normal Form game, and extend the results through a structural model to test the stationary distribution of research labs for different scenarios. The agents forming the foundation of this framework are: the Firms driving research; the Human Capital who perform the research; the Government that subsidizes the research; and, Investors who demand returns of the Firms.

The literature is growing in terms of positive theories explaining the reduction of technological investment, but is nearly non-existent with respect to a normative model to explain the interaction between the agents responsible for driving technological change. We are informed by Phelps (1966), who first posited the existence of a "Golden Rule" level of investment for Firms and Human Capital, but made no indication that a failure may occur for a competitive market equilibrium. [2] Our paper makes a contribution toward understanding this market failure and the mechanisms through which we can achieve a Socially Optimal level of technological investment and innovation.

In fact, Jones and Williams (1997) calculate the lower bound on the Social Return on Research and Development (R&D) as 30%, and that R&D is actually under-invested in by a factor of four below the Socially Optimal level. [3] We explore a normative model to understand this market failure, where the Government raises its subsidy level to match the increasing complexity and costs of performing research, in order to keep the returns on innovative, pre-commercial projects above the risk-free rate, thus stabilizing the rate of research within an economy.

Expounded in later section, increasing complexity makes research today more difficult than yesterday, i.e. the marginal cost of innovation is increasing over time. For example, a biologist exploring bacteria through a microscope today cannot perform their research without the help of two other specialists, from two separate fields: the electrical engineer to create the microscope, and a computer scientist to analyze the observed data. Another simple way to illustrate this point is to think about how millennia ago, the first Bakers of bread would have made their own ovens out of stone and firewood. However, today,

a Baker cannot make their own oven, but rather pays a team of engineers and business people at General Electric to develop and construct their ovens – a feat that is no longer apart of the Bakers’ skill set, because of increasing complexity over time.

What can a Government, acting as the Social Planner, do to keep the Solow-residual of  $A$  growing? First, we assume technologies are based on Pre-Commercial research (PCR). This is reasonable as technology builds on previous discoveries, and many previous discoveries may not have immediate commercial value. Second, the more uncertainty surrounding revenue generation from PCR, the less likely Firms are to invest. Third, a reduction in PCR investment will lead to a slow down in research. Thus, we propose a normative model, where it is the Social Planners responsibility, i.e. the Government, in the interest of increasing aggregate output, to invest in PCR such that we can continue to increase  $A$ .

The class of models developed in this paper is based on a Normal Form Coordination game between the Firm, which is the originator of R&D, and Human Capital, the PhDs and researchers of the economy without whom the Firm cannot perform research. The Investor acts as Nature, by demanding a rate of return that is required of Firms in order to keep investing in their projects. Finally, the Government observes the payoffs of the Firm and Human Capital, and the required market rate of return by Investors, and then chooses an endogenously-determined level of subsidy to Firms and Human Capital. The resulting equilibrium of the one-shot game is a function providing the required subsidy rate of the Government, in order to sustain an equilibrium where not only do Firms and Human capital both decide to dedicate themselves toward innovation, but satisfies the rate of return that Investors require of firms engaging in R&D projects with stochastic payoffs.

Our normative framework helps identify and fix this market failure, where the Government increases over time, its subsidy level to match the increasing complexity and costs of performing research, in order to keep the returns on such projects above the risk-free rate, and stabilize the number of research firms and rate of research within an economy.

## 2 Literature Review

### 2.1 Background and Historical Context

There is a vast literature about the relationship between technology and economic growth dating back to the early twentieth century. However, the literature has no accepted normative theory for how technological progress actually occurs, nor does the literature give a definite answer as to why the black-box of innovation, the Firm, may or may not choose to engage in basic research that leads to technological progress.

Joseph Schumpeter first used the phrase “creative destruction” to describe the economic process of technological renewal, where a more advanced tech-

nology would take the place of an older, less productive technology. [4] In the 1950's, Robert Solow

implemented a Cobb-Douglas Production function to model the growth of economies using three factors: capital  $K$ , the sum of factories, machines and other durable goods that are the amount of capital available in the economy; the total labor supply  $L$ ; and,  $A$ , a constant multiplier that represents the technological productivity of the economy. [5] While  $K$  and  $L$  are both econometrically observable, Solow admitted having no means of observing  $A$ . [6] Rather he said that since aggregate output  $Y$ , was also observable, he could calculate the last unknown  $A$ .

Solow's insight prompted economists to look more deeply into how technology and its effect on productivity help economies grow. When Solow was writing his landmark paper, he made several critical assumptions about  $A$ . First, that  $A$  over relatively long intervals of time, e.g. decades, is roughly constant and exogenously determined. This means that the multiplying effect that  $A$  provides to the equation grows slowly, even as labor and capital levels fluctuate throughout time. Second, in countries with similar capital and labor levels, there must be a factor, hypothesized to be technology, which causes incomes between countries to grow at different rates. As Abramovitz (1956) said in a scathing rebuttal to Solow, the "Solow residual" term  $A$ , was no more than a measure of our ignorance for how technology actually factors into the equations – his statement was mostly correct. Total Factor of Productivity (TFP) growth was taken as exogenous. Although Solow did not fully understand what affected the final value of  $A$ , he knew that Governments must look beyond increasing their populations and capital stocks in order to stay on a steady growth trajectory. However, most of the literature extending the neoclassical model, even until recently, focuses almost entirely on the dynamics of physical capital, with little to say about human capital or the Industrial Organization (IO) responsible for economic development and innovation. [7] However, Acemoglu and Dell (2010) recently found that roughly half of inter-country and intra-country income differences can be explained by differences in human capital level. Informed by this, the focus of our model is that Human Capital is the active agent of economic growth.

From Solow's work, economists found themselves with equally intriguing positive and normative questions to be explored over the next half-century. First, how does the presence of technology actually affect economic growth? Second, can policies influencing the level of technology in an economy affect growth?

Most of the economic growth and technology literature deals with the positive question. On the heels of Solow, others broke ground on both of these questions. Arrow (1962) furthered the Positive theory, supposing the concept of "learning by doing". Arrow said that the level of technology in Solow's model is a function of knowledge that could be acquired and accumulated by agents in the economy, and this was consistent with his use of the Cobb-Douglas production function. Addressing the normative question, Arrow (1963) asks how markets can function under the existence of information asymmetry between buyers and

sellers. If a seller of a good, technology or service has information, what market mechanisms can a Buyer use in order to feel secure about participating in the market. [8] We later explore this dynamic by Aghion and Tirole, using an IO framework.

Phelps (1964) pushes the normative possibility a step-further, saying that like capital, technology has a “Golden Rule” investment level and, at this steady state, a certain amount of the labor force should be dedicated to research. [9] The idea of a steady state investment level and a requirement for labor force participation in research will form the foundations of our model, to be described in the next section.

For two decades, the idea of how technology feeds back into economic growth remained dormant until Romer (1986). Romer created an “endogenous growth theory”, where technology is assimilated into the productive capacity of an economy through the accumulation of Human Capital – a formalized version of Arrow’s “learning by doing”. Romer’s model also offered the possibility for increasing returns to scale, and a normative steady-state growth path where a social planner may subsidize capital accumulation to incentivize a private economy to move toward a socially optimal level. [10] Two of Romer’s breakthroughs, critical to our model, are that 1) the endogenous accumulation of knowledge is a labor augmenting factor that provides increasing returns to scale on production, and 2) given externalities in this type of economy, capital subsidies to the private market can help the economy reach a socially optimal, balanced growth path. While it seems obvious that education can help make an economy more productive, what is not obvious is how private market subsidies may be required to boost an economy to the socially optimal growth level. Or more generally, to what extent can policies beyond general capital subsidies to the private market affect economic growth?

Robert Lucas (1988) famously said about questions of economic growth that “once one starts to think about them, it is hard to think about anything else”. Lucas augmented upon Solow and Romer’s two input growth models (labor and capital), by adding a third factor, “human capital” or H. Human capital is different than just “labor” (L), because in order to apply technology in an economy, labor must be skilled. Thus, L was re-labeled unskilled labor, and H skilled-labor. By adding human capital, Lucas found the affect of A on the outcome to be smaller than previously thought, and better explained the post-WWII growth in the developed world.

From the improved model, Lucas offered economists another challenge: we have successfully reduced the impact attributed to the Solow-Residual, but how do we calculate H? In the same paper, Lucas proposes two means of calculating H (in his attempt to exogenize the endogenous factor A). First, H grows as a function of exogenous education taken on by the laborers. Accumulating human capital is done only by accumulating education, which is actually an investment trade-off that laborers must make for themselves based on the market, cultural, institutional, and other forces. This model helps to better account for average rates of growth, but does not explain the whole distribution of growth across countries. Second, Lucas re-introduces the idea of “learning by doing”, and

that  $H$  can be accumulated by on the job training and the implementation of new technology within an economy. This model says that because differences in human capital arise not just from education, but from the ability of countries to acquire technologies, then human capital may grow at substantially different rates across countries, accounting for the diversity in income levels.

The issue is that neither model accounts simultaneously for both learning by doing and education. The subsequent literature, as well as our model, incorporates both education specific factors and spillover from “learning by doing”, to help us explain patterns in technological progress.

Lucas (1988) sparked a major revival in the literature on economic growth. For the next twenty years, academics will evaluate what exogenous and endogenous factors compose the Solow residual. The 1990s were a fruitful time for many theories on how technological progress affects economic growth, and what in turn can affect technological progress.

Barro & Sala-i-Martin (1991) create a model for technologically driven growth where they explain why the value of  $A$  varies between countries, but still cannot create a ground up version of calculating  $A$ , or a normative theory affecting  $A$ . [11] The framework assumes that there are two types of countries: Innovation Leaders and Laggards. Innovation Leaders improve their technology level through R&D, pushing the Technological Frontier. On the other hand, Laggards improve their technology and productivity by copying innovation already discovered by Leaders. Since Laggards can copy technology at a rate that is faster than a Leaders ability to develop new technology, the income among Laggards will grow at a faster rate – eventually converging. Our paper uses this idea that innovation is easier for those further from the Technological Frontier, and as countries and Firms approach the frontier, the increasing complexity and cost of research reduces the rate of innovation and growth. Also, since we still cannot explicitly measure  $A$  on an absolute scale, our best chance to measure  $A$  is in terms of a relative scale, where all other countries are benchmarked to the higher Leader value of  $A$  (Maddison, 1982).

This is an important fact when looking at China, which has achieved a large amount of its economic growth by “free-riding” from previously developed technologies instead of innovating their own. While industrial parts of China have seen rapid income “catch up” and convergence toward First World levels, they will face a slowdown unless they start trying to compete on the innovation frontier as well.

Aghion-Howitt (1992) model how human capital and market driven “vertical innovation” leads to higher technological levels. [12] Drawing on Romer (1986) and Lucas (1988), the A-H model says that there are three primary forces driving  $A$  in different directions. First, the desire to improve technology and market competitiveness is driving research forward, especially the possibility of securing a monopoly on such innovation through a patent. In our model, we incorporate a multiplier on revenue that is a function of how well patent and IP rights are secured in a given economy. Second, on the other hand, the fear of outdated a currently profitable technology, although potentially less advanced, is holding back innovation (an example of this is the battery industry, which has found

no incentive to drive innovation forward because their current business model is very profitable). We see this idea iterated by Goettler and Gordon (2011) where competition in the semi-conductor industry has actually led to less innovation, because Firms are afraid of outdated their own products.[13] Later in this paper, Gordon's structural analysis informs the class of models that we develop through this framework. Third, the accumulation of knowledge from education, technological spillovers, and learning-by-doing all affects economic growth. Our model focuses primarily on education and spillovers.

The AH model makes several modifications to the original Solow-Lucas model. First, technological innovation comes from a subset of well-educated laborers who transform intermediate goods into final goods by advancing productive technologies in Firms. Second, given an initial technological level, we can calculate the time between major leaps in economically viable innovation, and thus how fast  $A$  will grow. Again, the calculation of  $A$  is relative, with a growth rate based on an initial level.

In an effort to better explain the AH model at the Firm level, different theories emerged about firm decision making and endogenous technological change between Mankiw et al and Grossman et al.

First, Mankiw-Romer-Weil's (1992) theory of Process Innovation explores how endogenous technological change in growth models never exactly specifies how the technology impact would be included, or where in the economy it would be felt. Instead, they simply show that in countries with similar technological levels, the neoclassical model does well in describing differences in per capital income.[14] In the MRW model, they attempt to control for  $A$  by looking at different production processes. They model technological advancement as an improvement in machines that resulted in improved labor productivity. The MRW model for "Process Innovation" says that technology actually helps people work more productively, and the machines are trivial without the human operator.

As mentioned before, "Knowledge accumulation" was a new concept introduced by Romer (1986, 1990) and Lucas (1988), where knowledge spillover is a part of human capital. With Process Innovation, instead of "production technology", the ultimate goal in a growing economy becomes "accumulation technology", where knowledge is a non-rival good. Also, the MRW model for Process Innovation uses Romer's concept of increasing returns to scale for ideas, and takes R&D as an upfront fixed cost. In our model, R&D is an upfront cost as well, but that cost is an endogenous function of increased complexity, and the distance a Firm is from the technological frontier.

When using this model to explain history, we see that the most income divergence took place during the 1800s, not during the post-WWII era as many people thought was a consequence of the uneven benefits of countries embracing newly developed computer technology.

Second, the Grossman-Helpman (1994) model focuses on "Product Innovation" instead "Process Innovation", implying that demand is what drives technology and a love of variety drives real income increases.[15] Thus,  $A$  can be understood by examining the types of products that an economy produces,

and the relative trade advantage gained. Grossman-Helpman was the first to introduce trade as a driving force in this type of growth model, using the framework that newly developed products needed new markets to be sold in, so they would be a good proxy for technological advantage. Our model assumes a closed economy, and takes the affects of trade and spillover as exogenously given.

While the Grossman-Helpman model gives us the same results as the MRW model described above, it makes the strong assumption that innovation is a deterministic process that can be regularly and accurately predicted. Our framework instead thinks of innovative progress as a distribution, and solves for a dynamic steady state that can be normatively established.

Each of the above general theories hypothesizes how technology is developed in Firms via the interaction of physical and human capital, but less is understood about the internal workings of the firm that affect technological progress or how Firms make decisions to perform R&D. We assume the Firm is the black-box where research for advancing technology is performed. Supposing that the Firm is the primary agent of technological advancement, we ask how are Firms induced to combine physical and human capital into technological breakthroughs? Also, despite the importance placed on technologically attributed competitive advantage, why are Firms recently investing less in R&D? The answer to this normative question will give us the first steps in what policy measures can be taken to induce the advancement of an economy's technological frontier.

When Firms make a choice to perform R&D, they are prioritizing the Firm's resources over other possibly profitable ventures. In the situation where Firms have more information about a product than their investors, they tend to exhibit unique behavior about their resource allocation decisions. R&D investment is a good example, because the possible outcomes about R&D are largely unknown to anyone outside the Firm for two reasons. First, the profitability of basic R&D is largely stochastic, as it is hard to predict whether the research will yield a profitable product. Second, R&D tends to be secretive, and Firms try to keep information confidential in order to minimize the threat of competitors and spillover.

As a result of asymmetric information within the Firm-Investor relationship that exists with R&D, Firms tend to exhibit three behavioral characteristics. First, they tend to use internal funds for Pre-Commercial Research (PCR) projects. Not issuing debt or equity to fund these projects means that the risk is internalized by the Firm.[17] Second, if external financing is required, a Firm would rather issue debt than equity.[18] This implies that the Firm require (of themselves) at least same rate of return on their R&D investment, as they might get from debt. This will serve as a lower bound in our model. Third, since Firm's often forgo many opportunities to fund R&D with equity, their risk-aversion leads them to only ever capitalize on a subset of possible R&D projects. Pindyk (1990) reinforces this point, where irreversible investments, like the sunk cost of investing in R&D projects with an uncertain payoff, are "especially sensitive to risk", whether that be future cash flows, interest rates, or the total cost of the venture.[19]

There are also organizational dynamics within the Firm that force decisions that may not be in the best investment choices of the Firm. While the Firm has “formal authority” over the decision of whether to invest in R&D, the “real authority” over such decisions may not be strictly internal to the company management.[20] Aghion and Tirole look at the principle-agent relationships within organizations to understand how those with formal decision making power (formal authority) end up transferring some of that power to others with “real authority”.

Aghion and Tirole highlight several factors that affect this dichotomy. For our purposes, we will think of the Investor and the Firm as part of one organization: the Investor as the Principle and the Firm as the Agent. The Firm must decide on whether to proceed with a Project, given the return demands of the Investor. We define a Project as a venture initiated by the firm that the Principle and Agent believe will lead to a profitable outcome. It is a reasonable assumption for the Investor to be the Principle, as they possess “formal authority” to decide what projects will get funded. Otherwise, if a Firm decides to proceed with a project seen as unfavorable to the Investor or profitability, then the Investor can punish the Firm by not investing. On the other hand, the Firm has the “real authority” to do any Project it chooses, and can ignore the threat of the Investor. We ignore this second possibility, by assuming that managers of the Firm are at least risk-neutral, so they probably would not make decisions that might lead the Firm to bankruptcy, in the most extreme case.

We assume that if there is an option between two Projects, given that each Project has the same mean return, the Investor will opt for the project that Second Order Stochastically (SOS) Dominates, as they want to reduce their risk profile while achieving the same returns. We also think it reasonable to assume that even if an R&D project was projected to provide the same returns as normal company activities, the R&D project would be SOS Dominated, because there is typically more uncertainty in the success of the project and future cash-flows, so the variance in the returns is larger.[21] Aghion and Tirole found several conditions where the Investor would be willing for the Firm to proceed with R&D Projects. First, Projects must be “sufficiently innovative” and the Principle would allow the Firm to decide, as the Investor does not have enough expertise on the subject.[22] This was the case for R&D firms in the post World War II era, where private labs were given control of their own projects and those investing had a hard time understanding the work being done, as the research far outpaced the dissemination of information on those projects.[23] The Firms were literally pushing the technological frontier.

On the other hand, Aghion and Tirole also note factors that cause a Firm to assume more “real authority”, within bounds. Since Firms are ultimately running the Project, they have a “large span of control” over the outcome.[24] Finally, although we have described the Investor as a representative investor, in reality, the Investor is actually many Investors, and many Principals. The presence of “multiple principals” means that the Firm assumes more “real authority” over decision making. However, the flexibility on the decision making

process is bounded by the returns that are expected of the Firm. We assume that a typical investor has two options for investing, Equities and Risk-Free Government Bonds. Investing in the Firm is a form of Equity investment, and the Investor only has reason to invest in the Firm, if and only if, the Firm is producing an average return that is larger than the Risk-Free rate. This takes the form of a Grim Trigger strategy for the Investor in a One-Shot Game. As we identified earlier, the Investor has the option to punish the Firm for bad performance, and if the performance falls before the Risk-free rate, then the Investor can withdraw funding, and place the funds in bonds. Thus, Firms may have sufficient “real power” to proceed with R&D projects at their leisure, but only up until the point (we define as an investment cliff) that the projects produce an average return that is greater than can be achieved by investing in risk-free bonds. This is analogous to the point made earlier, where Firms are more likely to invest in Projects by issuing debt rather than equity. Thus, the average return-on-debt provides an appropriate lower bound for the return required of a Project in equilibrium.

Together Aghion-Hewitt (1992) and Aghion-Tirole (1998) suggest that changes in the rate at which technological breakthroughs are made can be approximated as function of investment (or lack thereof).

Thus far, we have five established facts. First, that technological progress is a critical driving force behind economic growth. Second, besides the accumulation of human capital through education or technological spillovers, the development of new technologies occurs within Firms. We define the Firm as either a one-person inventor workshop or a multi-national corporation dedicating resources toward R&D. Third, as shown by Romer, technological progress provides increasing returns to scale, so the multi-national corporation should have a higher payoff factor than the one-person workshop. The important consequence of this is that the one-person workshop, which can be thought of as an individual with a PhD who is specialized in research, is naturally deterred from performing independent research, especially as over time, they enjoy a smaller relative share as increasing returns to scale benefit larger firms. Fourth, future technologies are based on the current technology. Fifth, Firms have the option to invest in higher return PCR despite higher risk and the wishes of the Investor, but that freedom is bounded by the fact that the firm must produce returns that are higher than the risk-free rate.

These points leave us with two unanswered questions. First, if R&D has so much potential to be profitable and beneficial to overall economic growth, why are fewer firms over time engaging in PCR? Second, is there a normative solution to incentivize Firms to keep engaging in R&D?

## 2.2 Background and Present Day Context

The remainder of this paper is dedicated to answering the above two questions. First, we suggest that while R&D projects can be profitable, their profitability has decreased over time. Since new technological breakthroughs are based on previous discoveries, the overall complexity of technological research is increas-

ing over time, resulting in higher costs. Additionally, “previous discoveries” that result in commercial technological breakthroughs may not actually be profitable by themselves, but are a necessary condition for “radical” technological discoveries.[25] The idea of “technological ladders” informs the positive relationship we see between investment and TFP growth.[26]

An example of Pre-Commercial Research, or PCR, is the ability to inject a mixture of solvents into the ground. However, that ability alone may not be commercially useful until someone combines the injection method with drilling technology to get Shale Fracking for Natural gas recovery.[27] Thus, additional PCR is required before arriving at commercially viable R&D over time, which increases the time horizon and reduces the profitability of a potential project. Second, although policies cannot raise the potential revenue for an R&D project with an uncertain outcome, there are normative solutions for increasing profitability, like reducing the costs of R&D projects. Since PCR is the earliest stage of research, making it the most risky, least profitable, and furthest in time from a commercially viable product, the Government can use public funds to subsidize the cost of PCR for Firms. In fact, Jones and Williams (1997) calculate the lower bound on the Social Return on R&D as 30%, and that R&D is actually under-invested in by a factor of four below the Socially Optimal level that Romer hypothesizes.[28] Additionally, Quah (1998) shows empirically that R&D investment is critical to the economic growth, and that the availability of PCR (even if it is protected) is also essential, as it helps to assemble new technologies from old ones.[29] We develop a normative model to fix this market failure, where the Government increases over time its subsidy level to match the increasing complexity and costs of performing research, in order to keep the returns on such projects above the risk-free rate, and stabilize the rate of research within an economy.

Currently the Government is already engaged in subsidies in R&D on many direct and indirect levels, but these subsidies have actually been decreasing over time. For example, Firms do not train their own workers. Instead, they rely on Government funded PhD programs to train the research labor force. NIH and NSF grants have also been critical to professional researchers in funding their projects and the availability of funding for these grants has been decreasing as well, in a per application basis.[30] Lastly, the Government has been providing loans and guarantees to companies participating in innovative research, but these programs are small relative to demand.[31] Unless the Government increases the nominal rate at which it funds these programs in the face of rising costs, the rate of return from the subsidy will continue to fall in real terms.

### **2.3 Literature and Historical Evidence informing the Framework’s Foundations**

The increase in the cost of performing research is two-fold. As mentioned before, increased complexity means that more PCR is required to create the next technological breakthrough. Additionally, increasing Institutional costs play a major role in performing research.

Period	Electrification, Post Industrialization	Networked Computing, Post-Elect.
Year	1900	1999
Place	Curie Lab, Paris, France	Langer Lab, Cambridge, USA
# of Researchers	1.5	~130
# Discoveries/Year	1	~50
Variable Cost / Disc.	1-3x	3-5x
Fixed Cost per Year	3-6x	100x

Figure 1: Table comparing the per capita costs of research output at productive labs. Sources: Curie Notebooks; Nobel Prize Speech, 1908; global prices of Radium and ore in early 20th century.

To understand increasing complexity and institutional costs we use a thought experiment. In 1900, Marie Curie earned a Nobel Prize in Chemistry through her PhD thesis, by distilling the element Radium from rock ore.[32]

The process of distilling was done in a laboratory that lacked a water tight roof, and most of the manual labor was performed by Curie herself, or occasionally her husband Pierre. The ore was cheap by today's standards to acquire, and the instrumentation to perform her experiments were completely handled to Curie. Today, in a one of the most productive labs in the world, the Robert Langer Laboratory for biochemical research at MIT, there are state-of-the-art ventilation systems and facilities, and electron microscopes that a biologist could not hope to operate without the assistance of an electrical engineer or technician, let alone develop. The increased level of specialization over time is the prime illustration of how complexity, and the increasingly multi-disciplinary nature of research, has caused prices to rise. For research today, it does not require one Curie, but rather three specialists, from three separate fields: a biologist to identify the subject under the microscope, the electrical engineer to create the microscope, and a computer scientist to analyze the observed data. Another simple way to illustrate this point is to think about how millennia ago, the first bread Bakers would have made their own ovens out of stone and firewood. However, today, a Baker cannot make their own oven, but rather pays a team of engineers and business people at General Electric to develop and construct their ovens – a feat that is no longer apart of the Bakers' skill set.

In the past, the overall price or cost of research has been abated by demand from the Government and help from capital markets. The US military in the Cold-War era was a major patron of PCR and other scientific research.[33] While our model makes no assumptions about the initial existence of a demand side for PCR, it is important to note that US technological superiority did not arrive from a vacuum. Even the NIH and NSF have reduced the average funding they provide for projects over the last 20 years.[34] Additionally, before the 1980s, R&D Firms could utilize capital markets in such a way to raise funds

for projects.[35] The end of the Cold-War caused a “crowding in” of governments in capital markets, as newly industrializing states started issuing large amounts of debt, and real long-term interest rates rose.

In order for technological progress to be made at a socially optimal rate to advance economic growth, the Government may act as an agent subsidizing the costs of Firms in performing research. When taking on the role of the Social Planner, the Government observes the actions of Firms, Investors and Human Capital. From the Firms, the Government observes the current capacity for R&D research and the structure of how incentives affect the Firm’s R&D output. From the Investors, the Government observes the current rate of return demanded by Investors of Firms, which can inform the magnitude of the subsidy that the Government should be making to maintain R&D programs. For the Human Capital, and in the spirit of Phelps (1964), the Government can observe the current cost of research labor, as that may also inform the magnitude of the subsidy that is required, and how subsidizing training programs will indirectly reduce the cost of human capital by increase supply.

Most recently, in the past 15 years, economists have been looking more closely at the organizational structure of Firms and markets in order to understand the dynamics of innovation and development. This literature informs the environment in which our model is built. Goettler and Gordon (2012) demonstrate how market forces can either spur or impeded innovation for durable goods like semiconductors. They evaluate the innovative and consumer surplus outcomes that result from innovation being driven by either a monopoly or duopoly market. Unlike many papers on innovation, they do not assume that innovators have a perfect monopoly, as is theoretically afforded by a Patent. Thus, they proceed with their model, as we do, assuming the existence of a competitor pushing the technological frontier. They found that in a non-Monopoly market, Firms were disincentivized to innovate at the technological frontier, because the spillover between firms actually outweighed the costs of innovation. On the other hand, a duopoly actually innovates faster than a monopoly market, under the condition that there is rapid market growth.[37] This is consistent with what we highlighted before: in the past, innovation took place because the Government was a major market maker during the Cold-War on innovative technologies, effectively inducing rapid market growth, so PCR was profitable. In our model, profitability is the driving force behind Firm decisions, whether to innovate or otherwise. Atkison and Burnstein (2011) reinforce that subsidies affecting Firm profitability are essential in inducing Firms to perform PCR.[38] Since the end of the Cold War, the rate of PCR falls, controlling for costs, because the demand side is not what it once was. Without a Cold War, in order for the Government to sustain the profitability of PCR, that only leaves us with government policies to manage the costs of PCR. The need for the Government to help manage rising costs is also highlighted by Besanko et al (2010), where they agree that firms are helped to innovate through “learning-by-doing”, but firms also “forget” knowledge, so the Marginal Cost of production and innovation falls at a slower rate than it would without forgetting.[39] In our model, Forgetting can be seeing within our Representative Firm in two ways, 1) as small-innovative start-ups

that die before they reach the market place, and 2) the increasing complexity of innovation, where unless many specialized researchers are working together simultaneously, innovation does not come to fruition. They say Forgetting within Firms and an industry is akin to “racing down and upward-moving escalator”.

Goettler and Gordon start by modeling their duopoly as a Normal Form Game between the two agents. Since the market for semiconductors is not rapidly expanding, because computers are a durable good that do not need to be replaced frequently, their model takes the form of a Prisoner’s Dilemma form, where the two companies are actually likely to slow down their research rates. Informed by this, we model the decision of agents to engage in innovation as a Normal Form game, but since we are not constrained by a market with only durable goods, our Normal Form game takes the shape of a Coordination game, where Firms and Human Capital are inclined to perform research but must both simultaneously decide (or be induced) to do so. By investing themselves in technological innovation, the economy achieves the highest rate of innovation.

The model of our economy assumes the existence of a non-monopolist, Representative Firm (RF) that allocates some part of its resources, on average, to PCR. We are comfortable making this assumption for three reasons. First, like Goettler and Gordon, we do not assume that firms are small relative to the market, unlike previous studies by Hopenhayn (1992) and Klette and Kortum (2004).[40] This enables us to construct our model as the proposed Normal Form game, which gives rise to a strategic Markov-perfect Nash Equilibrium.[41] The research allocation is the choice variable for the RF, and the higher the proportion of resources allocated, the higher the number of firms or amount of funds dedicated to research. The economy is actually made up of many Firms, but we use the RF to simplify our model. Second, from empirical work by Aghion et al (1997) and Aghion et al (2005), we know that the strongest incentives to innovate are not from post-innovation rents, but rather the difference between post-innovation and pre-innovation rents, i.e. the major increases in revenue that come only from risky PCR and an eventual major technological leap.[42] Aghion (2005) finds that when plotting the number of patents in an industry against the Lerner index, we see an inverted U-Shape that was first noted by Scherer (1967).[43] Third, Acemoglu-Cao (2010) says that all Firms within an industry, whether leading-incumbent and laggard-entrant, are poised to engage in radical innovation. According to basic Schumpeterian models, the laggard-entrant is the primary driver of radical innovation. However, in our model, any entity within our Representative Firm, composed of both Incumbents and Entrants, can be incentivized to engage in PCR, and are responsive to Government PCR subsidies.

While the idea of an RF is a strong simplifying assumption overlooking the diversity within an industry, it is very difficult to account for the idiosyncrasies between industries. This may be an area for further research. On the other hand, nearly all Firms enjoy an increase in their competitiveness by increasing their technological and research level, and it is reasonable to assume that the vast majority of firms operate under this belief.

In our model, the Normal Form Coordination game’s equilibrium, between

the RF and the Human Capital (HC), is affected by the presence of the Government and the Investor. The goal of the Government is subsidize research in such a way as to reduce the cost of PCR, thus making the market more competitive for both leading Firms in research and lagging Firms. When competition is higher, the Schumpeterian effect is more likely to dominate, and a larger fraction of innovation is being performed by laggard Firms with low initial profits, instead on concentrated to the few leading firms.[44]

To estimate the equilibrium outcome of the Normal form game, we examine the effect of coordination on the innovation outcome, and estimate the preferences of HC and the economic payoff to the RF.[45] Our model's approach is consistent with those of Needham (1975) and Lee (2005), who similarly utilize consumer preferences and firm competencies as key determinants of R&D.[46] The consumer in this case is the HC who wants to consume research goals in order to satisfy themselves. It is safe to assume that if HCs have already endured a PhD program, their utility must be driven in some part by a desire to do pending-research. The Firm's competency in this case is its ability to profitably perform research.

### 3 The Framework and Description of Agents

*These four agents are: the Investor (I), the Representative Firm (RF), Human Capital (HC), and the Government (GV).*

#### 3.1 The Firm

The "Representative Firm" (RF) is defined as a risk-averse agent, resembling the hierarchical, IO of a modern corporation, with the CEO as the lead decision maker. The RF is also a private market agent whose goal is to remain economically competitive and growing over an infinitely discounted timeframe. The RF enhances their utility by acquiring or developing productive, physical capital (in the form of intellectual property, innovation, PCR, "spare-parts", or commercial products). The acquire or develop this physical capital by acquiring Human Capital (HC), who are the skilled, specialized section of the labor force who perform tasks that lead to economic growth, to either 1) develop new productive capital (PCR or commercializable products), or 2) to incrementally advance the existing product portfolio. Revenue stochasticity, risk-aversion and discount rate pressures form RI all factor into the RFs decision making process.

The firm choice of investment in R&D is on a continuum,  $i_{RD} \in [0, 1]$ , where they either dedicate sufficient proportion of their resources to an innovative project or not.

The Payoff (Profit) Function for the Firm take the form:

$$\pi(\theta, \alpha, \rho, \sigma, F, H) = \theta\alpha - \rho + \sigma$$

To obtain a closed form for optimal investment, we follow Pakes and McGuire (1994) and constraint the innovation outcome  $\tau_j = w'_j - w_j$  to be either 0 or  $s$ , which is one innovative step forward. If a company has stopped pursuing innovation, then they will revert to 0.

The Probability that the Firms makes an innovative step  $s$ , conditional on the current level of innovation and level of investment choice is:

$$\Pr_j(\tau = s|x, w_j) = \frac{a(w)^{*i_{RD}}}{1+a(w)^{*i_{RD}}}$$

Investment Efficiency is found given the frontier of technology  $\bar{w}$  :

$$a(w) = a(0) \max[1; a_1 \left(\frac{\bar{w}-w_j}{s}\right)^{1/2}]$$

Where  $a_1$  is the innovation spillover effect.

### 3.2 The Human Capital

The representative "Human Capital" (HC) is a risk-averse, skilled, specialized section of the labor force who perform tasks that lead to economic growth. Depending on what point in time and space we are observing, HC can either be predominantly at-home tinkerers, academics in a research lab, or scientists in a corporate research facility. The HC derives utility from a combination of financial return and "ability to produce new goods" (ie the joy of invention).

### 3.3 The Investor

The "Representative Investor" (RI) is defined as a risk-neutral agent that allocates resources toward Firms or Producers that yield the most potential return on investment. They are a Private market agent, and not affiliated with an Public sector institutions; or, much like how one would think of Adam Smith's Invisible Hand. The behavior of the RI can be observed by examining their portfolio of investments. The portfolio is composed of two classes of investments: either a risk-free bond, or an equity. We assume the RI is rational and risk-neutral. Thus, they will only invest in an equity position, if and only if, they see it as having a net return greater than the risk-free bond.

### 3.4 The Government

The "Government" (GV) which is a risk-neutral party, that has the ability to subsidize the cost of research. The GV derives its utility from a higher level of economic growth in the prevailing economy, which the government achieves by simultaneously encouraging both more labor and capital to be utilized by Human Capital and Firms, invest their resources toward PCR.

### 3.5 Rules, Notation, and Assumptions

### 3.6 Notation

$\delta$  : discount rate that is set by the RI, and affects the decision making of the RF.

$\sigma$  : subsidy on PCR as provided by the GV to reduce the cost.

$\rho_{PCR}$  : exogenous market price of engaging in PCR, that is increasing over time.

$\alpha$  : Baseline revenue for Firms.

$\beta$  : Baseline salary of HC, including opportunity cost of education, and specialization

$\omega$  : Salary Premium that HC receives for working in non-PCR occupation. This also factors in as a higher expense to Firms investing in PCR, who must pay higher wages to HC.

$\gamma$  : PCR spillover for Firms that do not invest in PCR, as a fraction of the CDT ( $\rho$ ). This rate is gradually decreasing, as firms and technologies reach the frontier of discover and known technology, and the markets saturate with discovered PCR. We can illustrate the change in gamma, but thinking of it in terms of the Shaving Industry. Let us say that there only exist two companies, and both companies make personal shaving equipment for Men. Company A manufactures a single razor blade, while Company B invents the be-all-end-all five-razor blade. Let us illustrate a very simple example the affect of gamma, "knowledge-spillover" over four time periods. In the first period, Company A already has a product with 1 blade, while Company B has a product with 5 blades, so we can think of gamma as the technological distance between one-blade Company A and five-blade Company B. If we assume that we can model this in a linear way, then the initial value of gamma is 0.8, because we think of it as  $\frac{\text{Be-all-end-all-5-blade technology} - \# \text{ of blades offered by Company A}}{\text{Be-all-end-all-5-blade technology}} = \frac{5-1}{5} = 0.8$ . Thus, Company A has yet to discover how to integrate 4 of the 5 blades into their product, which is analogous to 80% of the technology being not-yet-spilled over. Next period, Company A develops the two-blade technology, and then gamma reduces in value from 0.8 to 0.6 ( $= \frac{5-2}{5}$ ). Fast forwarding to the end of knowledge spillover, when Company A discovers how to fully implement the 5 blade technology, there is no technology gap between Companies A and B, and the gamma becomes 0 ( $= \frac{5-5}{5}$ ), because there no more knowledge to spillover.

$\theta$  : Intellectual Property Premium, the multiplier on base payoff that represents the value of intellectual property and social capital premium for engaging in PCR.

#### 3.6.1 Assumptions

A1. Technological change is a driver of economic growth.

A2. The priority of the Government is to advance policies that will drive economic growth, given an exogenous level of labor and capital. This is consis-

tent with Robert Lucas' assessment that to maintain stability in the economy, even during recessions, economic growth should be the primary focus.

A3. Factors  $\alpha, \beta, \omega, \gamma$  are time-independent and exogenous.

A4. Factors  $\rho, \delta$  time-dependent and exogenous factors.

A5.  $\delta$  is an exogenous time-discount factor that is determined by the RI, at a given point in time. We assume that the discount factor demanded by the RI has risen uniformly over time.

A6. Without government intervention, we typically see that  $\omega > \sigma$ . Or, the private market compensates HC at higher rates than the public market.

## 4 Model 1: The Normal Form Game

The Normal Form game and payoff structures are below. The game is has two agents as players, RF and HC, and two agents as outside parties of influence, RI and GV.

The payoff structure for the two players has the following intuition for each of the four possible outcomes. Each player must choose from two strategies: "Invest resources in PCR", or, "Not Invest resources in PCR". For the Firm, this means making a strategic decision to invest in PCR instead of trying to rely on knowledge spillover. For HC, this means deciding whether to work on PCR, or to work at another Firm that does not engage in PCR, but may have a higher financial and opportunity-cost compensation.

### **Four possible outcomes for (Firm - RF, Human Capital - HC):**

*(Invest, Invest)* : RF receives its standard payoff  $\alpha$ , plus a PCR-subsidy  $\sigma$ , but minus the cost of engaging in PCR  $\rho$ . HC receives their standard payoff of  $\beta$ , plus a PCR-subsidy  $\sigma$ , because PCR-subsidizing also increases their salaries.

*(Invest, Not Invest)* : RF receives its standard payoff  $\alpha$ , plus a PCR-subsidy  $\sigma$ , but minus the cost of engaging in PCR  $\rho$ , and minus the rise in HC wages because they are in lower supply in the PCR industry. HC receives their standard payoff of  $\beta$ , plus a premium  $\sigma$ , for working in a firm that is not engaging in PCR.

*(Not Invest, Invest)* : RF receives its standard payoff  $\alpha$ , but also receives PCR knowledge spillover at the rate of  $\gamma\rho$ . HC receives their standard payoff of  $\beta$ , but no PCR-subsidy  $\sigma$ , because the Firm is Not Investing, so they receive no PCR-subsidy.

*(Not Invest, Not Invest)*: RF receives its standard payoff  $\alpha$ , but no knowledge spillover at the rate of  $\gamma\rho$ , because HC is not investing themselves in PCR that will result in such spillovers. HC receives their standard payoff of  $\beta$ , plus a premium  $\sigma$ , for working in a firm that is not engaging in PCR.

The two other agents, RI and GV, affect the discount factor,  $\delta$ , and the PCR-subsidy,  $\sigma$ , respectively.

P1: Firm (RF) \ P2: Human Cap. (HC)	Invest	Not Invest
Invest	$(\theta\alpha - \rho + \sigma), (\beta + \sigma)$	$(\alpha - \rho + \sigma - \omega), (\beta + \omega)$
Not Invest	$(\alpha + \gamma\rho), (\beta)$	$(\alpha), (\beta + \omega)$

Figure 2: Fully-populated Normal Form Representation.

P1: Firm (RF) \ P2: Human Cap. (HC)	Invest	Not Invest
Invest	$(\alpha(\theta - 1) - \rho + \sigma), (\sigma)$	$(-\rho + \sigma - \omega), (\omega)$
Not Invest	$(\gamma\rho), (0)$	$(0), (\omega)$

Figure 3: Reduced Normal Form Representation. The Independence of Utility origins allow us to make this simplification without loss of generality.

#### 4.0.2 Procedure for Play

The game takes the form of a Coordination game, where the GV is attempting to sustain an equilibrium solution of (Invest, Invest). Theoretically, the GV will be simultaneously maximizing the sum of the utilities.

1. Given:

- time-independent, exogenous factors:  $\alpha, \beta, \omega, \gamma$
- time-dependent, exogenous factors:  $\rho, \delta$

2. The GV must choose a level for  $\sigma$ , such that over an infinite-repeat time frame, the GV sustains an equilibrium solution of (Invest, Invest) that theoretically maximizes the sum of the utilities, and the technological growth within the country.

#### 4.0.3 Characterizing the Equilibriums using One-Shot Deviation

We ask can a  $\delta$  discount rate be found (that is determined by interaction between Investors and the Government), that can sustain (Invest, Invest) as a (pure-)equilibrium strategy?

Since the payoffs are not symmetric, we need to derive two separate, necessary equilibrium conditions that correspond to Human Capital and Firms, separately.

Human Capital Decision to Not Deviate:

$$\begin{aligned} \frac{1}{1-\delta}(\beta + \sigma) &\geq \beta + \omega + \frac{\delta}{1-\delta}(\beta + \omega) = \beta + \omega\left(\frac{1}{1-\delta}\right) \\ \implies \\ \text{(E.1)} \quad \sigma &\geq \omega \end{aligned}$$

Firm Firms Decision to Not Deviate:

$$\begin{aligned} \frac{1}{1-\delta}(\theta\alpha - \rho + \sigma) &\geq (\alpha + \gamma\rho) + \frac{\delta}{1-\delta}(\alpha) \\ \implies \\ \frac{\delta}{\gamma\rho} &\geq \alpha + \rho(1 + \gamma) - \sigma - \theta\alpha \\ \implies \\ \text{(E.2)} \quad \delta &\geq \frac{\alpha(1-\theta) + \rho(1+\gamma) - \sigma}{\gamma\rho} \end{aligned}$$

These necessary conditions give us several points about observations within our model.

First, that the necessary condition for Human Capital is not dependent on a time-discount factor. This can be interpreted as consumers making choices for their next job with a sense of immediacy, or rather, that people choose their occupation based on what they want to do right now. Another way to see this is that, HC to "Not Invest" would be a Strictly Dominant strategy. This means that regardless of the shape that Firms decide to take, the amount of payoff that HC receives is greater if they allocate their personal human capital potential toward an occupation that is not in PCR. Thus, the only factor that can change this balance is for GV to increase the level of  $\sigma$ .

Second, in (E.2) the discount rate that prevents firms from deviating is decreasing in  $\gamma$ . We expect that as the cost of PCR falls, then the discount rate required on the market for firms to engage in PCR will be higher, and the rate required by investors ( $1-\delta$ ) will be lower, because of higher expectations of output and returns. Said another way, the marginal cost of capital used for innovation is  $1-\delta$ . [47] Third, similarly, we see that as  $\rho$  decreases, the discount rate rises. As the spill over rate decreases, we expect that the discount rate to also rise, because firms will not be able to rely on spillover to advance their internal research. Thus, the discount rate required is higher, which means that the cost of capital must be lower ( $1-\delta$ ) to sustain the equilibrium.

Fourth,  $\theta$  must be greater than 1, or else firms may see negative returns on their intellectual property. This emphasizes the importance of IP protection, and in economies where there is little IP protection, we can expect the  $\theta < 1$ . In the case of these types of economies, where theft of IP is common, the numerator shrinks, and the value of  $\delta$  is lower. This in turn results in a higher risk on the investment, and we see a higher rate of required return ( $1-\delta$ ) by investors.

Fifth, a slightly counter intuitive result, an increase in  $\sigma$  (the government subsidy on PCR) results in an increase in the required rate of return by the market. Since an increase in the  $\sigma$  results in a decrease in the numerator, then  $\delta$  also decreases. As  $\delta$  decreases, then the required rate of the market, seeing that PCR-research is more accessible, will demand a higher rate of return ( $1-\delta$ ). On the other hand, although raising the level of PCR subsidy  $\sigma$  is a necessary condition for (HC: Invest) to be a Weakly Dominated strategy, it is not sufficient

to guarantee the (Invest, Invest) equilibrium outcome. Additionally, subsidy  $\sigma$  needs to be at a high enough level, such that "Invest" is a Strictly Dominant strategy for the Firms (RF).

Sixth,  $\delta$  is an increasing function of both  $\rho$  and  $\gamma$ . The reason is that as the cost of PCR  $\rho$  rises, we can expect that firms will have a more difficult time performing research, and so  $\delta$  rises, and the rate demanded by the market is  $(1-\delta)$  goes down. This is the same with spill overs  $\gamma$ , when we have more spill overs, the equilibrium discount rate that firms demand of themselves goes up, because it is easier to develop technology, and the market will demand less of a return  $(1-\delta)$ , as they expect their investment to be less risky. On the other hand, there is intuitive importance around the inequality  $\sigma > \rho + \gamma\rho$ . Since  $\rho + \gamma\rho = \rho(1 + \gamma)$ , this means we can think of spill-over as a premium on the current cost of PCR, which actually incentivises Firms to not engage in PCR, as they are comfortable with the notion that someone other agent will perform the PCR for them. As mentioned above,  $\gamma$  is actually decreasing over time, which means the model matches our intuition that, as more knowledge spills over, there will be less remaining PCR to spill over, and the payoff to an RF for "Not Investing", decreases over time.

We observe an '*Investment Cliff*' when the  $(1-\delta)$  actually goes below the risk free rate of bonds! This is why there are so few profitable, public companies like Lucent that perform research. They die off!

In summary, we have the sufficient condition, if both  $\sigma > \omega + \rho$  and  $\sigma > \rho + \gamma\rho$  are true. For (Invest, Invest) to be a certain outcome, GV must raise the level of  $\sigma$  high enough such that both HC and RF choose "Invest" as their Dominant Strategy.

#### 4.0.4 How to maintain an equilibrium in an infinitely repeated game?

We assume that the Government has "dynamic" control of  $\sigma$ , and the Social Planners goal is to adjust  $\sigma$ , such that two conditions are satisfied:

(E.1.a.) If  $\sigma \geq \omega$ , then a infinite-period normal form game can sustain an equilibrium strategy of (Invest, Invest).

(E.2.a.) If  $\sigma$  is dynamically chosen each period by the Social Planner, as a function of  $\alpha, \theta, \rho, \gamma$ , then an (pure-strategy) equilibrium can be sustained, if the  $\delta$  is such that the market rate of return is higher than the risk-free rate.

(E.2.a) implies that the Social Planner must choose a level of  $\sigma(\alpha, \theta, \rho, \gamma)$  as a function of the four observable factors, such that the discount rate that Firms require of themselves (based on their expectations they see of the market) must be low enough, such that it clears risk free rate. A basic example is that, if the risk free rate is 0.05, then the discount rate for  $\delta$  that the firms see themselves requiring to be a market player, must be  $\delta < 0.95$ .

## 5 Conclusion and Further Research

This paper is meant to stimulate discussion about a new class of models that inform our understanding of how market and non-market agents interact to explain the changes in innovation and R&D investment policy, and why it might be pervasively below the Social Optimal level. While we chose a specific set of agents, ie Firms and Human capital, as our primary illustration for this paper, this set of models is generalizable to involve any number of other agents.

Firms, which are the bed of innovation and development, are decreasing the amount of time and effort they dedicate toward R&D research for three reasons. First, the increasing difficulty and cost of innovation has caused Firms to scale back their investments in projects that have a stochastic payoff, like R&D. Second, this problem is further compounded by the fact that Human Capital, who are the researchers driving innovation within Firms, are similarly hesitant to dedicate their careers to R&D, not only because Firms are not committing to R&D, but other higher payoff opportunities may arise. These related effects cause a fragile equilibrium that can be sustained by an endogenously determined subsidy level, where the Government provides public funding for R&D.

This paper also proposes the foundations for further exploring this coordination game through a Baseline and Structural model, both outlined in Appendices 2 and 3, respectively. The Baseline model makes minimal structural assumptions, while in Appendix 3, we explore a class of models informed by Goettler and Gordon (2011). Due to limitations on available data, we may find that one sub-family of this class of models may be more conducive to empirical evaluation and methods.

For further research and evaluation, we run a series of empirically tests, for example using available World Bank data to measure the relationship between the amount of investment in R&D and the returns to productivity and patent applications that are filed by countries around the world.

## 6 Appendix 1: References

[1] H. Becquerel, *Compt. Rend.*, (1896); P. Curie and M. Curie, *Compt. Rend.*, (1898); (1899); M. Curie, *Rev. Gen. Sci.*, (1899); *Rev. Sci.*, (1900); Nobel Lectures, Chemistry 1901-1921.

[2] Phelps, E. S. (1966), "Models of Technical Progress and the Golden Rule of Research", *Review of Economic Studies*, Vol. 33, 133-145.

[3] Charles I. Jones & John C. Williams, 1998. "Measuring The Social Return To R&D," *The Quarterly Journal of Economics*, MIT Press, vol. 113(4), pages 1119-1135.

[4] Schumpeter, "Capitalism, Socialism and Democracy", 1942.

[5] Solow, R.M., "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, 1956, pp. 65-94.

[6] *Ibid.*

[7] Acemoglu, Daron; Dell, Melissa. 2010. "Productivity Differences between and within Countries." *American Economic Journal: Macroeconomics*, 2(1):

169–88.

[8] Uzawa, H., 'Optimum Technical Change in an Aggregate Model of Economic Growth', *International Economic Review* 6 (1965), pp. 18-31.

[9] Phelps, E.S., 'Models of Technical Progress and the Golden Rule of Research', *Review of Economic Studies* 33 (1966), pp. 133-145.

[10] Romer, 1986, & 'Endogenous Technological Change', *Journal of Political Economy* 98 (1990), pp. 71-102.

[11] Barro, Robert J & Sala-i-Martin, Xavier, 1997. " Technological Diffusion, Convergence, and Growth," *Journal of Economic Growth*; Barro, Robert J & Sala-i-Martin, Xavier, 1992. "Convergence," *Journal of Political Economy*, University of Chicago Press, vol. 100(2), pages 223-51.

[12] Aghion P and Howitt P, 1992, "A Model of Growth through Creative Destruction", *Econometrica* 60(2): 323-351.

[13] Goettler, R. and B. R. Gordon (2011), "Does AMD spur Intel to innovate more?," *Journal of Political Economy*, 119(6), 1141-1200.

[14] N. Gregory Mankiw; David Romer; David N. Weil. "A Contribution to the Empirics of Economics Growth", *The Quarterly Journal of Economics*, Vol. 107, No. 2. (May, 1992), pp. 407-437

[15] Grossman, G.M. and Helpman, E., "Innovation and Growth in the Global Economy", MIT Press, Cambridge, 1991 (a). & 'Quality Ladders in the Theory of Growth', *Review of Economic Studies* 58 (1991), pp. 43-61 (b).

[16] Myers, Stewart C.; Majluf, Nicholas S. (1984). "Corporate financing and investment decisions when firms have information that investors do not have". *Journal of Financial Economics* 13 (2): 187–221.

[17] Ibid.

[18] Myers, 1983.

[19] Pindyck, "Irreversibility, Uncertainty, and Investment", 1990.

[20] Aghoin, Tirole, "Formal and Real Authority in Organizations", 1997.

[21] *Microeconomic Theory* (with Michael Dennis Whinston, Jerry R. Green, Oxford University Press, 1995.

[22] Aghion-Tirole, 27.

[23] Vannevar Bush, "Science, The Endless Frontier", 1945.

[24] Aghion-Tirole, 28.

[25] Olsson, O. (2005). "Technological opportunity and growth". Unpublished Manuscript, Göteborg University.

[26] Grossman, Helpman, 1991.

[27] Trembath, Alex, Jesse Jenkins, Michael Schellenberger, William Nordhaus, "Where the Shale Gas Revolution Came From", 2012, Breakthrough Institute.

[28] Jones, C.I. and Williams, J.C., 'Measuring the Social Return to R&D', *Quarterly Journal of Economics* 113 (1998), pp. 1119-1135..

[29] Quah, D.T. and Keely, L.C., 'Technology in Growth', CEP Discussion Paper 391 (1998).

[30] NSF, NIH Database: <http://grants.nih.gov/grants/oer.htm>.

[31] Small Business and Financing Act, 2009.

[32] Sources: Curie Notebooks; Nobel Prize Speech, 1908; Global prices of Radium and Ore in early 20th century. Table comparing the per capita costs of research output at productive labs.

[33] Ruttan, Vernon, "Sources of Technical Change", 1996.

[34] NIH, NSF Website Data, Table. This is controlling for the fact that there has been a large increase in the amount of proposals submitted to these institutions.

[35] Soete, L., 'Uncertainty and Technological Change: Discussion'. In: Fuhrer, J.C. and Sneddon, Little, J. (ed.), Technology and Growth: Conference Proceedings, Federal Reserve Bank of Boston, Boston, 1996, pp. 119-125. <http://www.bos.frb.org/economic/conf/conf40/conf40d.pdf>.

[36] Ibid, p.123.

[37] Ronald L. Goettler and Brett R. Gordon, "Does AMD Spur Intel to Innovate More?", Journal of Political Economy, Vol. 119, No. 6 (December 2011), pp. 1141-1200.

[38] Atkison and Burnstein (2011)

[39] Besanko et al (2010)

[40] Hopenhayn, Hugo A, 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, Econometric Society, vol. 60(5), pages 1127-50, September.

[41] Tor Jakob Klette & Samuel Kortum, 2004. "Innovating Firms and Aggregate Innovation," Journal of Political Economy, University of Chicago Press, vol. 112(5), p. 986-1018.

[42] Aghion, 1997, 2005.

[43] Scherer, Fredrick, "Market Structure and the Employment of Scientists and Engineers," *American Economic Review*, LVII (1967), 524 -531.

[44] Aghion, 2005, p.702.

[45] When firms compete on innovation, we can estimate consumer preferences and firms' innovation efficiencies, which determine the benefits and costs of innovation, and solve for equilibrium under various competitive scenarios. From Goettler, Gordon, 2012.

[46] Needham, D. (1975), 'Market Structure and Firms' R&D Behaviour', *Journal of Industrial Economics*, vol. 23, no. 4, pp. 241-55.

[47] Acemoglu, Dell, 2010, p.183.

## 7 Appendix 2: Framework for Baseline Model

We frame the problem on an infinite horizon where the Firm maximizes its pay-off. Unlike Model in Appendix 3, this most simple version makes minimum structural assumptions. There is only one agent, the Firm, and the cost of Human Capital is exogenously given, and included in the rising cost of innovation  $\rho$ .

$$v_t(\theta, \alpha, \rho, \sigma) = \max_{\{\text{Not Invest}; \text{Invest}\}} [\pi_0(1, \alpha, \rho, \sigma) + \beta v_{t+1}(\theta_{t+1}, \alpha_{t+1}, \rho_{t+1}, 0); \pi_1(\theta, \alpha, \rho, \sigma) + \beta v_{t+1}(\theta_{t+1}, \alpha_{t+1}, \rho_{t+1}, \sigma_{t+1})]$$

$\pi(\theta, \alpha, \rho, \sigma) = \theta\alpha_t - \rho_t + \sigma$   
 $\rho_t = \rho_0 \exp\{r_\rho t\}$   
 $a_t = a_0 \exp\{r_\alpha t\}$   
 $\sigma_t$  exogenously given.

HC's Utility Function take the infinite consumption sequence, from HC time-discount factor  $d$ , and follows the Inada Conditions of:  $u_{HC}(d, \bar{c} = \text{"Pure Strategy Payoff from the Normal Form"}) = \sum_{t=0}^{\infty} d^t c$

RI's Utility Function takes the general risk-neutral, discounted infinite-horizon form of:  $u_{RI}(\delta, u_{RF}) = \sum_{t=0}^{\infty} \delta^t u_{RF}$

## 8 Appendix 3: Framework for Structural Model for this Scenario

The model is informed by Goettler and Gordon (2011), where Firms solve the maximization problem subject to the operating costs of continuing to engage in a innovative projects. The Firms will stop operating said projects (unsustained) if the return of risk free is higher than the payoff from the projects. The result is a distribution of R&D projects that are sustainable.

The structural equation should be derived from the normal form equilibrium conditions. For this Structural Model framework, we reduce the potential outcomes above to an equilibrium where the Invest, Invest strategy is sustained for a non-zero number of Firms and Human Capital.

### 8.1 The Firm

The firm choice of investment in R&D is on a continuum,  $i_{RD} \in [0, 1]$ , where they either dedicate sufficient proportion of their resources to an innovative project or not.

In each time period, a Firm  $j$ , has the option to engage in R&D or not. To obtain a closed form for optimal investment, we follow Pakes and McGuire (1994) and constraint the innovation outcome  $\tau_j = w'_j - w_j$  to be either 0 or  $s$ , which is one innovative step forward. If a company has stopped pursuing innovation, then they will revert to 0.

The Probability that the Firm invests in an innovative step project, conditional on the current level of innovation and level of investment choice is:

$$\Pr_j(\tau = s|x, w_j) = \frac{a(w) * i_{RD}}{1 + a(w) * i_{RD}}$$

Investment Efficiency  $a(w)$  is found given the frontier of technology  $\bar{w}$ . The important point about this is that the closer a Firm is to the Technological frontier, the more complex an incremental innovation will be. Thus, the probability of have a commercially viable project come out of the Firm's investment decision is less.

$$a(w) = a(0) \max[1; a_1 \left(\frac{\bar{w} - w_j}{s}\right)^{1/2}]$$

Where  $a_1$  is the innovation spillover effect.

The Payoff (Profit) Function for the Firm take the form:

$$\pi_j(\theta, \alpha, \rho, \sigma) = \theta\alpha + \sigma - \rho_j$$

The Marginal Cost of Innovation at the representative firm, in country j, is:

$$\rho_j = \lambda_0 + \lambda_1(\bar{w} - w_j)$$

Thus, the Bellman for the Firm's maximization problem is:

$$F(\bar{w}, w_j, \theta, \alpha, \sigma) = \theta\alpha + \sigma - (\lambda_0 + \lambda_1(\bar{w} - w_j)) + \beta \sum F(\bar{w}', w_j', \theta', \alpha, \sigma') * \Pr_j(\tau = s|x, w_j)$$

We assume that  $\alpha$  is static in this model, because we make no assumptions about the revenue growth generated by the Firm, only that the profit is sufficiently high enough to please investors.

The only choice a Firm can do is decide whether to invest in Innovation or not. This means it resembles a search function form, since we cannot take the derivative of a binary variable.

To characterize the behavior of the solution, the firm will keep investing in innovation, until after a certain time period, if the payoff is not positive, or higher than the discount rate required by investors, given government subsidies, we will see the firms die off.

j is the number of firms within a country, and in our model.

## 8.2 The Human Capital

The representative "Human Capital" (HC) is a risk-averse, skilled, specialized section of the labor force who perform tasks that lead to economic growth. Depending on what point in time and space we are observing, HC can either be predominantly at-home tinkerers, academics in a research lab, or scientists in a corporate research facility. The HC derives utility from a combination of financial return and "ability to produce new goods" (ie the joy of invention).

The utility of the Human Capital is:  $u_{iR} = \beta + \sigma_{i(R)} + \omega_{i(1-R)} + \varepsilon$ , where  $\beta$  is the base pay,  $\sigma$  is the amount of government subsidy with  $y$  as the multiplier on the government subsidy,  $\omega$  is the premium that is received by the HC if they decide to not invest in research and instead go to Wall Street, and R is a binary variable indicating whether they invest themselves in research or not.

$$H(\beta, \sigma, \omega, R) = \beta + \sigma_{i(R)} + \omega_{i(1-R)} + \delta \sum H(\beta, \sigma', \omega', R') * \Pr_j(\tau = s|x, w_j)$$

## 8.3 The Investor

The "Representative Investor" (RI) is defined as a risk-neutral agent that allocates resources toward Firms that yield the most potential return on investment. While an Investor is likely to invest mostly in the top returning firms, we assume for simplicity that the market will Invest in any firm that promises a rate of return that is greater than the risk-free rate  $r_{Free}$ .

## 8.4 The Government

The "Government" (GV) which is a risk-neutral party, that has the ability to subsidize the cost of research by choosing  $\sigma$ . The GV derives its utility from a higher level of economic growth in the prevailing economy, which the government achieves by simultaneously encouraging both more labor and capital to be utilized by Human Capital and Firms, invest their resources toward PCR. However, the optimal choice of the Government is to choose a subsidy level  $\sigma$ , such that Firms are profitable enough, such that the discount rate of  $\delta$  required to sustain an (Invest, Invest) pure-strategy Nash Equilibrium is equal to  $1 - r_{Free}$ .

## 8.5 The Equilibrium

The proposed equilibrium is informed by Goettler and Gordon (2011), and results in a function  $\sigma(\delta, ..)$ , such that a Nash Equilibrium arises.

The equilibrium specifies two important criteria. First, that the Firm and Human Capital have rational expectations and their equilibrium strategies can on be functions of the current time period, which comprises all of the payoff relevant values. Second, that the Firm and Human Capital have rational expectations each others policy functions about whether to Invest or not Invest, and about the evolution of the distribution of future R&D projects.

The equilibrium is illustrated by running the MATLAB Script (Appendix 4). We find that without an increasing amount of subsidy for R&D, the Best Respond of Firms and Human Capital is to play a mixed strategy, that over time, places less emphasis on Investing in research and more emphasis on Not Investing (ie Deviating). In the long-run, the number of projects reaches a low steady state, where the amount of Firms and HC that Invest is minimal, and the R&D project that are Invested in, give a return that is at least at the risk-free rate. While this is an equilibrium, as first highlighted by Kwah (1998), this is far below the Socially Optimal Level.

The model predicts that sustaining a Socially Optimal Level of investment would require the Government to subsidize R&D at an increasing rate. However, we suppose that this is sustainable, because the increasing returns from aggregate technological progress will be higher than the cost to keep subsidizing the research.

## 9 Appendix 4: MATLAB Psudeo-code Developed for Project

```
% Norm_Setup.m: global parameter values for the Normal Form Problem  
%
```

```

%{
How is the Program Structured:
1. Setup.m
   Establishes Global Variables

2. Utility Functions, UtilityX.m
Both Utility Functions take the same form, but take in different parameters
for their payoffs to be considered.
This Scripts should do a one time utility valuation, given inputs from a
specific time period in the game.

The Payoffs to be considered

3. The Game Structure, Repeated Form with Outputs.

4. Environmental Factors
How does the Environment Change each period

%}

global IPPrem_Theta FirmBase_Alpha RDSub_Sigma HCBASE_Beta HCPrem_Omega Spillover_Gamma;

IPPrem_Theta =1;
FirmBase_Alpha =1;
%RDCost_Rho =1;
RDSub_Sigma =1;
HCBASE_Beta =1;
HCPrem_Omega =1;
Spillover_Gamma = 10;

%UtilityFirm.m: utility function for Firm Payoffs

%Define the Function
function UtilityFirmOutput = UtilityFirm(D_FirmInvest,D_HCInvest,RDCost_Rho)
%The 3 inputs are: the Firm Decision, HC Decision, and exogenously changing
%R&D costs

global IPPrem_Theta FirmBase_Alpha HCPrem_Omega Spillover_Gamma;

if (D_FirmInvest == 1) && (D_HCInvest==1)
    UtilityFirmOutput = IPPrem_Theta*FirmBase_Alpha - RDCost_Rho + HCPrem_Omega;
elseif (D_FirmInvest == 1) && (D_HCInvest==0)

```

```

        UtilityFirmOutput = FirmBase_Alpha - RDCost_Rho - HCPrem_Omega;
elseif (D_FirmInvest == 0) && (D_HCInvest==1)
        UtilityFirmOutput = Spillover_Gamma * FirmBase_Alpha + RDCost_Rho - HCPrem_Omega;
else
        UtilityFirmOutput = 0;
end;

```

% UtilityHC.m: utility function for retirement (employment) problem

```

%Define the Function
function UtilityHCOuput = UtilityHC(D_FirmInvest,D_HCInvest)
%The 3 inputs are: the Firm Decision, HC Decision, and exogenously changing
%R&D costs

```

```

global RDSub_Sigma HCBase_Beta HCPrem_Omega ;

```

```

if (D_FirmInvest == 1) && (D_HCInvest==1)
        UtilityHCOuput = HCBase_Beta + RDSub_Sigma;
        %UtilityHCOuput = 1 + 1;
elseif (D_FirmInvest == 1) && (D_HCInvest==0)
        UtilityHCOuput = HCBase_Beta + HCPrem_Omega;
elseif (D_FirmInvest == 0) && (D_HCInvest==1)
        UtilityHCOuput = 0;
else
        UtilityHCOuput = HCPrem_Omega;
end;

```

% ExpUtilityFirm.m: EXPECTED utility function for Firm problem

```

%Define the Function
function UtilityFirmOutput = ExpUtilityFirm(D_FirmInvest, Prob_HCInvest, RDCost_Rho)

```

```

if (D_FirmInvest == 1)
        UtilityFirmOutput = (Prob_HCInvest)*UtilityFirm(1,1,RDCost_Rho) + (1-Prob_HCInvest)*Util
else
        UtilityFirmOutput = (Prob_HCInvest)*UtilityFirm(0,1,RDCost_Rho) + (1-Prob_HCInvest)*Util
end;

```

% ExpUtilityHC.m: EXPECTED utility function for Human Capital problem

```

%Define the Function
function UtilityHCOuput = ExpUtilityHC(D_HCInvest, Prob_FirmInvest)

```

```

if (D_HCInvest == 1)
    UtilityHCOutput = (Prob_FirmInvest)*UtilityHC(1,1) + (1-Prob_FirmInvest)*UtilityHC(1,0);
else
    UtilityHCOutput = (Prob_FirmInvest)*UtilityHC(0,1) + (1-Prob_FirmInvest)*UtilityHC(0,0);
end;

%Simulate.m: program to simulate the data, and choices of agents (Firm and Human Capital Wit
%

Norm_Setup; %This is to initalize all global variables

%Set Time Horizon
%For most of our models, we will be looking at the post-WWII era, over a 50
%year time span
FirstState = 0;
LastState = 50;

%Intiliaz Output Dataset
data = [];

%The Goal of the FOR loop is to document the evolution in the behavior of
%Firms and Human Capital, over the time span

%What are the steps for the simulation:
%1. Each Firm and HC calculates their Set of Payoffs
%2. Each Firm and HC calcualtes their Expected Payoffs
%3. Each Firm and HC evaluates their Best Response
%4. Each Firm and HC decides whether to invest or not

RDCost_Rho = 1;
Exog_RD_Increase = 1.01;
TimePeriod = FirstState;
SigmaFirm = 1;
SigmaHC = 1;
BR_ProbHC =1;
BR_ProbFirm =1;

tic;
%LOOP: each period.
    for t=FirstState:LastState;    %Loop: For each round
        TimePeriod = TimePeriod + 1;

        %{

```

```

This first step is for diagnostic purposes

FirmPayoff11 = UtilityFirm(1,1,RDCost_Rho);
FirmPayoff10 = UtilityFirm(1,0,RDCost_Rho);
FirmPayoff01 = UtilityFirm(0,1,RDCost_Rho);
FirmPayoff00 = UtilityFirm(0,0,RDCost_Rho);

HCPayoff11 = UtilityHC(1,1);
HCPayoff10 = UtilityHC(1,0);
HCPayoff01 = UtilityHC(0,1);
HCPayoff00 = UtilityHC(0,0);

ExpFirmPayoff11 = ExpUtilityFirm(1,1,RDCost_Rho);
ExpFirmPayoff10 = ExpUtilityFirm(1,0,RDCost_Rho);
ExpFirmPayoff01 = ExpUtilityFirm(0,1,RDCost_Rho);
ExpFirmPayoff00 = ExpUtilityFirm(0,0,RDCost_Rho);

ExpHCPayoff11 = ExpUtilityHC(1,1);
ExpHCPayoff10 = ExpUtilityHC(1,0);
ExpHCPayoff01 = ExpUtilityHC(0,1);
ExpHCPayoff00 = ExpUtilityHC(0,0);
%}

ExpFirmPayoff10 = ExpUtilityFirm(1,BR_ProbFirm,RDCost_Rho);
ExpFirmPayoff01 = ExpUtilityFirm(0,BR_ProbFirm,RDCost_Rho);

ExpHCPayoff10 = ExpUtilityHC(1,BR_ProbHC);
ExpHCPayoff01 = ExpUtilityHC(0,BR_ProbHC);

%Evaluate Best Response decisions and Mixed Strategy outcomes
%This will take the form of a standard logit
%The best response for player A, is 1/(...)
%where
BR_ProbFirm = (exp(ExpHCPayoff10/SigmaFirm))/(exp(ExpHCPayoff10/SigmaFirm)+exp(ExpHCPayoff01/SigmaFirm));
BR_ProbHC = (exp(ExpFirmPayoff10/SigmaHC))/(exp(ExpFirmPayoff10/SigmaHC)+exp(ExpFirmPayoff01/SigmaHC));

%Exogenous Change in Cost of R&D
data = [data; [TimePeriod RDCost_Rho BR_ProbFirm BR_ProbHC ExpFirmPayoff10 ExpFirmPayoff01]];

%Updates in the Loop
RDCost_Rho = RDCost_Rho * Exog_RD_Increase;
end;

save('data','data');
SolveTime = toc

```