Risk Aversion and Expected-Utility Theory: A Calibration Exercise*

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Abstract

Rabin (2000) argues that, under expected-utility, observed risk aversion over modest stakes implies extremely high risk aversion over large stakes. Cox & Sadiraj (2006) have replied that this is a problem of expected-utility of wealth, but that expected-utility of income does not share that problem. We combine experimental data on moderate-scale risky choices with survey data on income to estimate coefficients

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of relative risk aversion using expected-utility of *consumption*. Assuming individuals cannot save implies an average coefficient of relative risk aversion of 1.92. Assuming they can decide between consuming today and saving for the future, a realistic assumption, implies quadruple-digit coefficients. This gives empirical evidence for narrow bracketing.

1 Introduction

According to expected-utility theory, risk aversion arises due to the concavity of the utility function. This explains aversion to large-scale risks but implies people are approximately risk neutral over modest stakes.[1] Rabin (2000) and Rabin & Thaler (2001) argue that, under expected utility theory, the levels of risk aversion we observe over modest stakes would imply absurdly high levels of risk aversion over large stakes. Cox & Sadiraj (2006) critique this argument due to the fact that Rabin considers expected utility of terminal wealth. They show that the same implications do not arise when considering a model of expected utility of income.[2]

In this paper we look at neither expected utility of wealth nor of income, but rather use an expected utility of consumption model to estimate coefficients of relative risk aversion. When looking at commonly used microeconomic theory textbooks, both Varian (1992) and Nicholson (2002) introduce utility as a function defined over consumption. Mas-Colell, Whinston &

[1] This is true for concave differentiable utility functions.

[2] For other critiques see Palacios-Huerta & Serrano (2006), Rubinstein (2001), and Watt (2002).

Green (1995) first introduce utility as a function defined over a 'set of possible alternatives' but soon thereafter they state these are alternative consumption bundles.[3]

We use survey data on income and experimental data on bet choice in a risk game to calculate rural Paraguayans' coefficients of relative risk aversion. If we assume the player must spend all his winnings in one day, we estimate reasonable coefficients. The assumption of no savings (or 100% depreciation) is equivalent to the assumption that consumption each day equals income from that day (in this case, the day on which the games were played). Nevertheless, this assumption is obviously faulty, as a player could choose to save his winnings rather than spending them all at once.

We then make the alternate assumption that the player decides in each period how much of his income to consume and how much to save (either with or without earning interest). Thus we take into account the net present value of the individual's income/consumption stream, not just his income and consumption at one point in time. When we assume the player can save, the coefficients calculated are in the quadruple digits and are quite unreasonable. Allowing for savings and taking into account the future income stream is similar in philosophy and yields similar numerical results to looking at expected utility over wealth.

With no savings, the experimental bet is a relatively high stakes bet (since consumption is equal to income on that one day) and so plausible coefficients

[3]Searching for utility in wikipedia, one finds the entry begins with the following sentence: "In economics, utility is a measure of the relative happiness or satisfaction gained by consuming different bundles of goods and services."

of relative risk aversion arise. But, when players can save, the experimental bet becomes a relatively moderate stake bet, implying absurdly high levels of risk aversion. Given values of risk aversion which are considered plausible in the literature, the evidence in this paper suggests that farmers do not take into account their wealth or their future income stream when evaluating each risky decision, thus rejecting the hypothesis of asset integration.[4,5]

This is the first paper that we know of to estimate risk aversion from economic experiments taking into account data regarding players' income or wealth levels, regardless of whether utility is defined over wealth, income, or consumption. The rest of the paper is organized as follows: Section 2 discusses the literature measuring risk aversion as well as the literature on narrow bracketing as an explanation for the unreasonable coefficients of relative risk aversion often found. Section 3 describes the experiment and data from rural Paraguay while Section 4 lays out how we estimate coefficients of relative risk aversion with expected utility of consumption under the assumptions of no saving, saving, and additional background risk. In that section we also estimate coefficients of relative risk aversion with expected utility of wealth. Section 5 concludes.

[4]Under asset integration utility is derived from final wealth rather than gains and losses.

[5]It could also be the case that players view the experiments as detached from the rest of their life. After the games one farmer stated he was going to go get drunk because his wife had no claim over his winnings. He explained that this was because the money did not come from his salary but his own "special winnings."

2 Discussion

Binswanger (1981) found that choices made over modest stakes by rural Indians lead to implausible coefficients of relative risk aversion 25 years ago, although he did not incorporate individual income or wealth. His calculations of *partial* relative risk aversion (the coefficient of absolute risk aversion multiplied by the size of the gamble rather than wealth, a concept developed by Menezes & Hanson (1970) and Zeckhauser & Keeler (1970)) yield reasonable coefficients. On the other hand, if he assumes asset integration he derives coefficients of (Arrow-Pratt) relative risk aversion four to six digits long. He does not use data on individual wealth to calculate these coefficients, but instead assumes that all players have a wealth of 10,000 rupees (while modal wealth in the village is 13,000). He calculates that, holding the player's choice constant, an increase in certain gains in the gamble of 100 rupees decreases the coefficient of absolute risk aversion as much as would an increase in wealth of 10,000 rupees. This causes him to reject asset integration.

At first these results may seem in contrast with much recent experimental research calculating coefficients of relative risk aversion which claims to find single-digit coefficients. This research ignores the fact that players may save their winnings or may come into the game with some level of initial wealth. One of the most well-known of these is work by Holt & Laury (2002) finding average coefficients of approximately 0.4 when defining utility over gains, not wealth. Cardenas & Carpenter (2007) review coefficients calculated ignoring wealth from risk experiments in developing countries and find estimates between 0.32 and 1.25. When using mean annual income as wealth (assuming people can not save from year to year) double-digit coefficients are found from deductible choice in the Israeli car insurance market (Cohen & Einav 2006) and high single-digit coefficients from play in a TV game show (Gertner 1993).

An innovation in this paper is that we use data on both income and choices over risky prospects to calculate players' coefficients of relative risk aversion. Economists generally think that wealthier people are less risk averse. Holt & Laury (2002) find that "income has a mildly negative effect on risk aversion" and Schechter (2007) finds wealthier people are less risk averse. Can the fact that wealthier people may choose more risky gambles lead to more plausible coefficients when taking into account individual wealth than the quadruple-digit coefficients found by Binswanger (1981) when he assumed every player had the same wealth level? In this paper, even after matching individuals' incomes with their moderate-stakes bets, we still calculate unreasonably large risk aversion parameters.[6]

If accounting for individual income does not lead to reasonable coefficients, then perhaps players are evaluating risky decisions in isolation. This is sometimes called 'narrow bracketing' or 'narrow framing'. Using choices made by contestants in multiple rounds of a TV game show, Gertner (1993) gives evidence that players segregate risky decisions. Read, Loewenstein & Rabin (1999) discuss the many ways choice bracketing may affect decision making in daily life. Kahneman & Lovallo (1993) posit that people are overly timid in their choices because they evaluate risky prospects one at a time rather than pooling risks. Even the equity premium puzzle brought to

[6]In the data section we discuss the advantages of using income rather than wealth to derive consumption.

light by Mehra & Prescott (1985) has been explained by 'myopic loss aversion' (Benartzi & Thaler 1995) in which investors are both more sensitive to losses than to gains *and* evaluate their portfolios frequently. More recently, Fudenberg & Levine (2006) have proposed a dual-self self-control model in which a short-run self is risk averse only over pocket cash while a long-run self is risk averse over wealth.

3 Data and Experimental Design

We surveyed 223 rural households in Paraguay in 2002.[7] All households who participated in the survey were invited to send one household member to participate in economic experiments and 188 chose to do so. The rules of the risk game were as follows: the player was given 8000 Guaranies (two-thirds of a day's wages) and could choose to bet nothing, 2000, 4000, 6000, or all 8000 Guaranies. The experimenter then rolled a die to determine the player's payoffs. A roll of one meant the player lost his bet, two meant he recovered only half his bet, three meant he recovered his bet, four meant he earned 1.5 times his bet, five meant he doubled his bet, and six meant he earned 2.5 times his bet.

Throughout most of this paper, when calculating the coefficient of relative risk aversion for each player we use his income although we do have

[7] In 1991 the University of Wisconsin and the Centro Paraguayo de Estudios Sociológicos implemented a survey of 300 rural Paraguayan households in three departments and sixteen villages across the country. The sample was random, and stratified by land-holdings. The original survey was followed up by subsequent rounds in 1994, 1999, and, the data we are using, 2002.

a measure of physical wealth (the value of land, animals, and tools owned by the household). We derive consumption using income rather than wealth because we cannot measure the value of human capital. Farmers with more education may make more profitable production choices, farmers with less land may take on day labor jobs, and some households include a teacher or a nurse earning wage income. Since income is the returns to all capital, not just physical capital, we use income in our calculations rather than our measure of physical capital itself. For comparison, we do also calculate coefficients of relative risk aversion using data on wealth.

4 Estimating Risk Aversion Parameters

4.1 Risk Aversion Under the Assumption of No Saving

At first let us assume that earnings are not risky. Later we will extend the analysis to incorporate background risk. Players could choose to bet nothing, 2,000, 4,000, 6,000, or 8,000 Guaranies on the roll of a die. We calculate the lower bound on the coefficient of relative risk aversion assuming a player who bet 4,000 Guaranies was just indifferent between betting 4,000 and 6,000. For those players who bet all 8000 Guaranies, we assume that they are not risk loving, so the lower bound on their coefficient of relative risk aversion is 0. Before incorporating background risk, the player's daily returns to wealth equal the household's annual income divided by the number of adult equivalents in the household multiplied by 365. We assume that the player will not share his winnings with other members of his family. As family size is not a significant predictor of the bet in the risk game, this assumption does not seem egregious. We divide all monetary quantities by 1,000.

Assuming constant relative risk aversion (CRRA) CES utility ($U(c) = \frac{c^{1-\gamma}}{1-\gamma}$) and that participants are not able to save, utility is defined over daily consumption (daily income plus winnings from the risk experiment). Thus, income and consumption are identical, and so models of expected utility of income and expected utility of consumption yield identical results. The average lower bound on the coefficient of relative risk aversion calculated in this way is 1.92. If we define utility only over winnings from the risk experiment and do not incorporate daily income from outside the experiment, we derive an average lower bound of 0.81. This is in line with the other studies summarized in Cardenas & Carpenter (2007) which estimate risk aversion from experiments in developing countries ignoring income outside the experiment.

4.2 Risk Aversion with Saving

We know that players do not necessarily consume all of their winnings in one day. They may save some of their winnings in a bank account or simply store it in their pocket. A player chooses optimally how much of his winnings to consume and how much to save for a later date given the interest rate and his discount factor. Let us first assume that, if a player saves, he earns the interest rate given by Crédito Agrícola de Habilitación, the main lender to small-scale farmers. This is 17.5% annually, or .0442% a day with compounding. Assume that players earn a constant daily income of y. We define the following variables: c is the player's consumption, s is savings, R is one plus the daily interest rate, w is his winnings in the risk game, and β

is the discount factor.

The player's problem when he can save is

$$V(w) = \max_{s_0}[U(y + w - s_0) + \beta V(s_0 R)]$$

and the Euler equation is

$$U'(y + w - s_0) = \beta R U'(y + s_0 R - s_1).$$

Assuming CES utility this implies

$$(y + w - s_0)^{-\gamma} = \beta R(y + s_0 R - s_1)^{-\gamma}$$

which means $c_t = \psi^t c_0$ where $\psi = (\beta R)^{\frac{1}{\gamma}}$. We can solve for the value function

$$V(w) = \left(\frac{1}{1 - \psi^{(1-\gamma)}\beta}\right)\left(\frac{c_o^{1-\gamma}}{1-\gamma}\right). \tag{1}$$

The intertemporal budget constraint is

$$c_0 = \left(w + \frac{y}{1 - \frac{1}{R}}\right)\left(1 - \frac{\psi}{R}\right) \tag{2}$$

Plugging equation (2) into equation (1) we get our final solution:

$$V(w) = K\left(w + \frac{y}{1 - \frac{1}{R}}\right)^{1-\gamma}\frac{1}{1-\gamma} \tag{3}$$

where $K = \left(\frac{1}{1-\psi^{(1-\gamma)}\beta}\right)\left(1 - \frac{\psi}{R}\right)^{1-\gamma}$.

We use equation (3) to calculate how risk averse players of different income levels must be to choose each bet. Remember that, for those players who bet all 8000 Guaranies, we assume a lower bound on their coefficient of relative risk aversion of 0. For those who did not bet all 8000, we know their daily income, y. Beginning with a coefficient of relative risk aversion of $\gamma = .01$, we use the value function to determine how much such a player should bet. If he bet less than he 'should', we increase γ by .01 until he switches to betting the amount he bet rather than betting the next highest option. This gives us the lower bound on his level of risk aversion.

Assuming the participants can save, the average lower bound on the coefficient of relative risk aversion rises to an absurdly high 2428. Previously, when we assumed participants could not save, the result of 1.92 seemed fairly reasonable. Allowing for savings, the numbers seem quite large. In addition, the few players with very high incomes have astronomical coefficients. Note that with $R = 1.00042$, $V(w) = K(w + 2382y)^{1-\gamma}\frac{1}{1-\gamma}$. A player who can save chooses the same gamble as a player 2382 times richer who cannot.[8]

Looking at equation (3) we see that while R (one plus the interest rate) will matter for the determination of the coefficient of relative risk aversion, β (the discount factor) will not.[9] One might argue that very few Paraguayans have access to savings accounts which offer a 17.5% annual interest rate. They may save their money by putting it under their pillow or investing in livestock with much lower returns. Assuming a lower interest rate only serves to increase the implied coefficient of relative risk aversion. As the interest rate goes to zero ($R \to 1$), the coefficient of relative risk aversion for someone who does not bet all 8000 guaranies goes to infinity. Mathematically this is because, as the interest rate goes to zero, the optimal bet goes to infinity. Any person who does not bet the maximum of 8000 Guaranies must be infinitely risk averse.[10] The intuition behind this is that when an individual

[8] Thank you to Edi Grgeta for pointing out this interesting multiplier effect.

[9] Although, if $\beta R \neq 0$ then consumption will either go towards 0 or infinity over time.

[10] Rather than considering a bet on the roll of the die, consider the simpler case of a

earns interest on his savings then the difference between winning a little and winning a lot becomes magnified. When the individual can save but earns no interest, the difference in net present value between winning a little and winning a lot is much lower. Thus, for the saver who earns no interest, the bet is a relatively lower stake bet than for the saver earning a high interest rate.

4.3 Risk Aversion with Background Risk

Gollier & Pratt (1996) show that all CRRA utility functions are risk vulnerable, meaning that adding mean-zero background risk will lower the optimal investment in any other independent risk. The fact that we have assumed no background risk may lead us to estimate inappropriately high coefficients of relative risk aversion. Although we do not know income variability, even if we assume that a farmer with daily income y has a 50/50 chance of earning daily

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bet on the toss of a coin. Imagine a player decides how much to bet, b, and has a 50/50 chance of winning the high amount hb or the low amount lb where $h > 1 > l > 2 - h$. From equation (3), the player's maximization problem is

$$\max_b V = \frac{1}{2}K\left((h-1)b + \frac{y}{1-\frac{1}{R}}\right)^{1-\gamma}\frac{1}{1-\gamma} + \frac{1}{2}K\left((l-1)b + \frac{y}{1-\frac{1}{R}}\right)^{1-\gamma}\frac{1}{1-\gamma}.$$

Solving the first order condition we find that $b = (y/(1-\frac{1}{R}))[(1-l)^{-\frac{1}{\gamma}} - (h-1)^{-\frac{1}{\gamma}}]/[(h-1)^{\frac{\gamma-1}{\gamma}} + (1-l)^{\frac{\gamma-1}{\gamma}}]$. Thus, as $R \to 1^+$ we see $b \to \infty$ and so, no matter the level of risk aversion, a player should choose to bet the maximal amount. If the player does not, he must be infinitely risk averse. We also calculate the first and second partial derivatives and find that for $R > 1$, $\frac{\partial\gamma}{\partial R} < 0$ and $\frac{\partial^2\gamma}{\partial R^2} > 0$. This means that for a given bet size, the coefficient of relative risk aversion is increasing at an increasing rate as the interest rate decreases and approaches 1. In fact, as R goes to 1 from above, $\frac{\partial\gamma}{\partial R}$ goes to $-\infty$.

income .25y or 1.75y for the rest of his life (an extreme level of background risk) our previous results still hold qualitatively.[11] The average lower bound on the coefficient is 1.22 if the player must spend his winnings that same day and 608 if the player is allowed to save. This is considerably lower than the coefficient estimated without background risk (2428) but is still magnitudes higher than traditional estimates.

4.4 Risk Aversion under Expected Utility of Wealth

For the sake of comparison, it is interesting to calculate coefficients of relative risk aversion using an expected utility of wealth model. As discussed earlier, the survey includes detailed information with regards to the value of physical capital stocks but not human capital stocks so it underestimates wealth. Continuing to assume CES utility ($U = \frac{(\omega+w)^{1-\gamma}}{1-\gamma}$), utility is defined over wealth, ω, plus winnings, w. In this case, the average lower bound on the coefficient of relative risk aversion is 2062. Compare this to the slightly higher but quite similar estimate of 2428 when using expected utility of consumption and allowing savings with no background risk. Given that wealth is underestimated, one would expect risk aversion to be underestimated as well. This is a nice robustness check for the previous estimation.

[11]Since this risk is decided once, this situation is much riskier than one in which the farmer has a 50% chance each day of having high or low income on that day.

5 Conclusion and Implications

Rabin (2000) and Rabin & Thaler (2001) argue that expected-utility theory cannot explain decisions people make over both modest stakes and large stakes coherently. In this paper we use choices made in modest-stakes risk experiments by rural Paraguayans whose income levels are known and assume a model of expected utility of consumption. If farmers can save their winnings, which they surely can, the implied coefficients of relative risk aversion are absurdly high. On the other hand, the same coefficients calculated over gains rather than final income are quite reasonable.

We give empirical evidence rejecting asset integration. This suggests that players isolate the risky decision in the game from considerations of their final wealth status. Some other theory of choice such as Kőszegi & Rabin's (2006) reference-dependent preferences or Fudenberg & Levine's (2006) dual-self model, in which large and small stakes bets are either implicitly or explicitly treated differentially, may be necessary.

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