Imperfect Competition and Sanitation: Evidence from Randomized Auctions in Senegal*

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Abstract

We study the extent to which collusion can explain the under-provision of clean sanitation technologies in developing countries. Using desludging services in Dakar as a case-study, we document that prices are 66% higher in areas where prices are likely coordinated by a large trade association, compared to nearby neighborhoods supplied by unaffiliated companies. We then develop an experimental just-in-time auction platform with random variation in several design features aimed at learning about the extent of competition. Consistent with the collusion hypothesis, we find that most bidders systematically avoid competition by placing round bids and refusing to undercut rivals.

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1 Introduction

Imperfect competition resulting from coordination between firms is common in developing countries and leads to higher prices and fewer transactions. Coordination in pricing and the exertion of market power are important impediments to growth and efficient resource allocation: imperfect competition diverts business from lower-cost firms and allows high-cost firms to remain in the sector.[1] Market power can be particularly strong in developing country markets, where antitrust enforcement is limited, leading to high mark-ups (Barrett 1997, Bergquist & Dinerstein 2020). In the context of products and services that have positive externalities, the welfare costs of market power are even more severe (for example, Barkley (2022)). State procurement can be plagued with inefficiency and high prices (Best et al. 2019). Electronic procurement and intermediation through auctions may facilitate an increase in competition and a decrease in procurement prices, although thusfar such auctions have been found to primarily increase quality rather than reduce prices (Lewis-Faupel et al. 2016). We develop an experimental auction system for sanitation services with randomized variation in design features to test the extent to which market suppliers maintain collusive activities under different auction conditions.

We analyze the importance of imperfect competition in the market for sanitation services in Dakar, Senegal where the under-provision of clean sanitation technologies increases the risk of health problems. As a consequence of rapid urbanization and under-investment in public infrastructure, most peri-urban areas of Dakar and many other large cities in developing countries are not connected to a sewage network. Instead, households rely on individual sanitation systems such as septic tanks and unimproved pits. These systems need to be emptied periodically (in Dakar this occurs on average twice per year); a service that we refer to as *desludging*. Households choose between two options: manual workers who enter the pit and extract the sludge using shovels and buckets and dump the sludge in the street; and truckers who pump the sludge into a tanker truck and take it out of the neighborhood, usually to one of three treatment centers. Survey evidence suggests that slightly more than half of desludgings in Dakar are performed using the manual option, which creates important environmental and health externalities including increased diarrhea incidence (Deutschmann et al. 2022).

The industrial organization of the market for residential sanitation services limits competition, potentially contributing to the low take-up of mechanized desludging in Dakar. Competition is limited in large part by the existence of a trade association (AAAS) controlling the prices set in the main garages where truckers park and meet residential clients.[2] We evaluate the importance of imperfect competition using a two-step approach. First, we conduct a non-experimental comparison between neighborhoods where the Association does and does not exert influence to study the Association's impact on price competition. Second, we generate experimental evidence by im-

[1] See Aghion & Griffith (2005) and Asker et al. (2019) for examples.

[2] Hereafter we refer to AAAS as the cartel or the Association.

plementing an auction platform with randomized invitations and auction format and analyze the bidding behavior of providers invited to participate in over 5,000 procurement auctions.

Our first approach to measure the effect of collusion on prices is to non-experimentally examine the difference in prices between neighborhoods controlled by AAAS and an adjacent municipality (Rufisque) dominated by unaffiliated companies. Prices in Rufisque are 40% lower than in the rest of the city, even after controlling for household and location characteristics. We document a steep price gradient outside of Rufisque as we move away from the border, suggesting that most consumers in the areas controlled by the Association do not have access to suppliers operating in Rufisque. As a result of this price difference, over 90% of the population in Rufisque uses mechanized desludging compared to 40% in the rest of the city. Although this is consistent with the presence of collusion in Dakar, we cannot rule out the possibility that companies in Rufisque are different from companies in the rest of the market along unobserved dimensions.

Second, to provide further evidence of collusion within Dakar, we use experimental data from an auction platform to construct a test of non-competitive pricing. Together with the government of Senegal, we designed a just-in-time auction platform for mechanized desludging jobs. The goal was to decrease prices and increase households' access to the improved sanitation technology. Households could contact the call center to obtain a quote for a mechanized desludging. Suppliers were invited to bid by text message, and the lowest bid was presented to the client, who could accept or reject it. The design of the platform included a number of experimentally randomized features aimed at measuring firms' propensity to compete. Jobs were offered to a randomly selected group of between 8 and 20 potential bidders. The auction format was also randomized. In half of the auctions, the platform used a revisable-bid format which periodically provided invited participants with information about the current lowest standing bid and allowed bidders to revise their bid. The other half of auctions were conducted using a sealed-bid format. The bidders' identities were not revealed to the other bidders in either format.

The experimental design allows us to determine whether providers are willing to defect from the cartel arrangement and make (secret) competitive offers in the auctions. If enough providers make non-competitive offers on the platform, it implies that the cartel is stable enough to prevent deviations from collusive prices even when the probability of detection is low. We identify potentially collusive strategies that are inconsistent with competitive bidding, or, equivalently, strategies that are inconsistent with individual profit maximization (Chassang et al. 2022*a*, Porter & Zona 1993, 1999). We identify different such strategies in revisable and sealed-bid auctions, and the randomization between auction formats allows us to observe differences in bidding strategies across the formats.

The first sub-optimal strategy which we document is the use of focal prices. Focal prices are common in the traditional market. In the auctions, bidding focal prices leads to a high probability of tying (20%), in which case the job is allocated to the participant who submitted the bid earliest.

There is a simple and strictly profitable deviation: Cutting one's bid by an imperceptible amount can increase the probability of winning by up to 20%. We interpret a bidder's high propensity to tie as evidence of his being part of a tacitly collusive agreement to soften price competition.

The second sub-optimal strategy is bidding early in the revisable-bid auction format. In this format, bidders are informed of the current lowest bid every 15 minutes, and have the option of submitting a sealed bid in the last ten minutes of the auction.

Assuming private costs, bidding before the closed portion of the revisable-bid auction rather than sniping in the closed portion of the auction is a sub-optimal strategy. Waiting reduces the likelihood that the bid is undercut and, to the extent that bidding is costly, firms are better off learning about rival bids before submitting their own bid in this paid-as-bid system. In contrast, submitting an early bid can be viewed as an effort to coordinate prices by sending a signal to rivals.

Our first set of results documents important differences in the distribution and timing of winning bids between the two auction formats. In particular, sealed-bid auctions are significantly more likely to end in a tie (10 percentage points), and revisable-bid auctions are significantly more likely to attract late winning bids (29 percentage points). In a significant number of revisable-bid auctions, a seller will wait until the end of the auction period and then undercut his rival in order to win. Despite these differences between formats, not all auctions appear to be competitive. In particular, 24% of sealed-bid auctions receiving more than one bid end in a tie, largely because firms heavily rely on commonly used focal prices (57% of sealed winning bids are on a 5,000 CFA grid). Similarly, a large share of winning bids in the revisable-bid format are placed early (58%).

Next, we exploit the panel dimension of our data to distinguish between collusion and other market frictions that could explain these sub-optimal strategies. In particular, we measure the persistence of firm conduct across auctions and confirm the existence of one group of bidders behaving competitively, and another group behaving non-competitively. There is a strong positive correlation between a bidder's propensity to tie in sealed-bid auctions and their propensity to submit a late bid (undercutting the standing low bid) in the revisable-bid auctions. In other words, competitive bidders avoid ties by relying less on focal prices and are more likely to bid late and undercut other bidders in the revisable-bid auctions, while apparently collusive bidders consistently avoid competition by submitting round bids and bidding early. We estimate that roughly 1/3 of active bidders are competitive types, while a majority of bidders appear to be colluding.

Tacit or explicit collusion is one of several reasons why firms may fail to maximize expected profits. Capacity constraints could be another explanation, but we document that most providers operate with substantial excess capacity. Pricing frictions caused by cash transactions such as the ease of paying with bills of specific denominations, can also limit the ability of firms to submit optimal bids. Although this type of friction is certainly present in this market, we show that changing the auction format (revisable vs sealed-bid) has a significant impact on the frequency of ties. We also document substantial heterogeneity in the use of round numbers across bidders, and

in particular find that bidders who are more likely to undercut or submit lower bids are also more likely to avoid ties. Both results suggest that bunching is at least partly related to firm conduct.

Alternatively, learning could explain firms' failure to use optimal strategies as in Doraszelski et al. (2018), although bidders were on average invited to bid in over 450 auctions, and we show that they quickly learned to behave strategically. Another potential explanation is bounded rationality and the difficulty of bidding optimally in complex auction environments as in Hortaçsu et al. (2019). While we cannot rule this out completely, we document that firms responsible for tying bids are also more likely to bid early and avoid under-cutting. We also find that sub-optimal bidding strategies are associated with other indicators of collusion commonly used in the literature. Apparently collusive firms submit higher bids on average, and are less likely to bid for clients located farther from their main garages (consistent with the presence of exclusive territories). Early bids in revisable-bid auctions are associated with significantly higher winning prices, relative to early bids in sealed-bid auctions. This is consistent with early bids serving as a coordination mechanism in revisable-bid auctions. We conclude that the presence of non-competitive bids is at least in part due to bidders' efforts to avoid competition, as opposed to being solely due to bidders learning, committing errors, or being inattentive.

Giving consumers access to competitive quotes can lead to large increases in the take-up of mechanized desludging. We estimate that changing the composition of invited bidders by inviting fewer collusive-types lowers the expected winning bid by 300 CFA (or 1.2% of the modal offer).[3] Similarly we find that receiving an early bid in a revisable auction (relative to a sealed-bid auction) on average leads to winning bids that are 750 CFA higher, confirming that bidder coordination has a meaningful effect on final prices. We believe this gives a lower-bound on the equilibrium effect of inviting more competitive bidders to the auctions. Very few auctions include more than one or two competitive bidders and so under the current random invitation rule competitive bidders do not need to place very low bids in order to win in a majority of auctions (this is the umbrella effect discussed by Caoui (2022)). If the platform targeted invitations to favor competitive bidders, these bidders might start bidding more aggressively.

This paper builds on an extensive literature testing for collusion. Following the work of Porter & Zona (1993, 1999), we define collusive behavior as a set of actions that violate individual profit maximization. Recent papers using a similar strategy to define collusive behavior include Chassang et al. (2022*a*,*b*), Clark et al. (2021), Conley & Decarolis (2016), and Kawai & Nakabayashi (2022). Our paper also relates to an extensive literature using excessive correlation in bids to detect collusive behavior (Abrantes-Metz et al. 2006, Bajari & Ye 2003, Froeb et al. 1993). There also exists a large literature studying the inner working of cartels, including Asker (2010) and Pesendorfer (2000).[4]

[3] This corresponds to a one standard-deviation decrease in the average "collusivity index" among active bidders invited.

[4] Our paper is also related to the broader empirical literature studying the behavior of cartels (Byrne & De Roos 2019, Clark & Houde 2013, Genesove & Mullin 2001, Igami & Sugaya 2022).

In contrast to those papers, our focus is on documenting the existence of non-competitive bidding as evidence of firms' collusive conduct.

Our findings also relate to the literature in industrial organization identifying behavioral biases and failures to maximize profits. DellaVigna (2009) provides an early survey of empirical findings (including in auction environments), and DellaVigna & Gentzkow (2019) documents the prevalence of sub-optimal pricing strategies in retailer markets. While we interpret our findings through the lens of tacit collusion, the presence of market frictions (e.g., cash-based transactions) and information frictions (e.g., biased beliefs) can contribute to the existence or appearance of strategic 'mistakes'.

The paper also contributes to a growing literature studying the importance and impact of market power in developing countries. Similar to our setting, Banerjee et al. (2019) conduct a series of procurement auction experiments to evaluate effect of outsourcing the last-mile delivery of rice in Indonesia. Their findings echo ours: outsourcing can lower profit margins when auctions attract a large enough number of competitive bidders (non-incumbent distributors in their setting). Bergquist & Dinerstein (2020) also uses a field experiment (in Kenya) to measure agricultural traders' market power, focusing on the the pass-through of cost shocks and subsidies to retail consumers.[5] Brown et al. (2022) use observational data on extortion payments to illustrate the importance of the organized crime gangs in sustaining price collusion in the wholesale market for food and pharmaceutical products in El Salvador. Barkley (2022) exploits the collapse of the pharmaceutical cartel in Mexico to measure the impact of collusion on patient health vis-a-vis the availability and cost of insulin. Additional papers measuring the consequence of imperfect competition in developing countries include Chilet (2018) (Chilean pharmacies), Neilson (2021) (Chilean elementary schools), Ryan (2021) (Indian electricity markets), and Walsh (2020) (Ghanaian radio broadcast markets).

Less work has been done on the impact of market power on sanitation markets, particularly in developing countries, yet the welfare impacts of poor sanitation are large. Given the high elasticity of demand for mechanized desludging of 2.2 estimated in Deutschmann et al. (2021), higher prices will cause substantially fewer households to use more sanitary technologies, particularly in poorer neighborhoods. Using mechanism design tools to improve sanitation markets and deliver more sanitary services to poor households has the potential to lead to great improvements in sanitation markets and health (Houde et al. 2020, Johnson & Lipscomb 2020). Houde et al. (2020) estimates the marginal cost of service providers consistent with observed bidding behavior in order to quantify the counter-factual benefit of offering a competitive auction platform to consumers.

The remainder of the paper is organized as follows. In Section 2, we describe the different data sources and background information on the mechanized desludging market. Section 3 gives non-experimental evidence of collusion in the traditional mechanized desludging market. Section

[5] Another study using a field experiment to study market power is Byrne et al. (2022), who conduct an audit-study to test for price discrimination in the Australian electricity market.

4 describes the experimental auction design and presents evidence of collusion using auction and bid outcomes. Section 5 discusses alternative explanations for the seemingly collusive strategies, provides further evidence that these strategies are in fact due to tacit collusion, and evaluates the effect of imperfect competition on prices and demand. Section 6 concludes.

2 Residential desludging in Dakar

When a household latrine pit fills, the household must empty it by having it desludged. Households have a choice between three types of desludging services: (i) manual performed by a family member, (ii) manual performed by a hired worker (called a "baay pell" in Senegal), and (iii) mechanized using a vacuum truck. Manual desludging consists of removing the sludge using a shovel and placing it in a pit dug in the street near the house.[6] Mechanized desludgings are done by two to three workers with a vacuum truck. The truck pumps as much sludge out of the pit as possible and either dumps it legally at a treatment center or illegally in a street drainage canal or the ocean.

The market for mechanized desludging is organized around three treatment centers scattered across the city and a network of garages (or parking lots) where clients meet service providers. Each operator typically belongs to one garage and parks their truck there between jobs while waiting for additional business. An important feature of the market is that prices are not posted but rather are determined by bilateral negotiation between the client and the truck driver. The proximity of consumers to garages therefore affects both their ability to search and negotiate for better prices, as well the cost of providing the service. Walk-in clients at a parking lot are allocated to the driver who is first in line, and drivers from the same garage do not compete over clients (similar to what often happens at taxi stands). Since this is a repeated business, clients often contact truckers directly; either by calling their phone number or hailing them on the street.

In this section, we start by describing the three main sources of data that we use to conduct our analysis. We then describe the structure of the market for mechanized desludging, and provide a series of stylized facts about supply and demand.

2.1 Data sources

The intervention and data collection spanned mid-2012 through mid-2015 in residential neighborhoods surrounding Dakar, Senegal. We collected (i) administrative data from just-in-time mechanized desludging auctions, (ii) a baseline and endline mechanized desludging provider survey, and (iii) four rounds of a household survey of desludging technology choice and price,.

Auction platform administrative data: We use panel data from a just-in-time auction platform for the procurement of residential desludging jobs. Together with Water and Sanitation for Africa

[6] The manual option increases the risk of health-related sanitation problems (for the client, the workers, and other households in the neighborhood). Manual desludging is technically illegal, though it is rarely sanctioned. It is often a source of controversy among neighbors.

(WSA) and the National Office of Sanitation in Senegal (ONAS), we ran the call center from July 2013 through September 2015. After that, the call center was scaled up by ONAS and then given to a private sector partner, Delvic. Call-center activity is clustered around the peri-urban areas of Dakar.

We have access to the administrative data from these auctions which include the randomized format of the auction, the randomized number and identities of the desludgers invited, whether or not they bid, the time and amount of their bid if they made one, the location of the household that they were bidding on, and the winning bid. The auctions are described in more detail in Section 4.1.

Survey of mechanized desludging service providers: We conducted a baseline survey of 121 desludging truck operators in mid-2012, and an endline survey of 152 drivers (of which 13 were owner-operators), 75 truck owners, and 20 managers in mid-2015. We tried to conduct a census of trucks active in the residential desludging market though our sample likely misses some independent truckers. An operator is either the manager of a fleet of trucks, or a driver associated with a single license plate. These two surveys include most operators who participated in the auction platform, as well as several who decided not to participate. The survey identifies each truck's main garage and other attributes.

Household survey: We have an unbalanced panel of 16,255 observations from 9,672 households. We drop observations from households that did not receive a desludging over the past year (either mechanized or manual) or with missing responses on key variables. The final sample includes 9,970 observations from 6,121 unique households. We use the household survey to measure the distribution of prices and demand across neighborhoods. An observation corresponds to a household's most recent desludging transaction performed in the 12 months prior to the survey. Since there are no posted prices, we use transaction prices reported by households for both mechanized and manual desludgings. To select households, we overlaid grid points on a map of Dakar, excluded any grid points which were in uninhabited areas or served by the city sewer network, and spiraled out from each starting point to select households. Appendix Figure B-1 displays a map with the households' locations. Our analysis focuses on seven of the 19 arrondissements of Dakar.[7]

2.2 Description of the market for mechanized desludging

Figure 1 shows the locations of the garages and the three treatment centers. On average, consumers are located 4 km away from the closest treatment center, 1.2 km from the nearest garage, and 2 km from the second closest garage. There is wide variation in the size of garages. The largest garage

[7]We chose residential arrondissements and avoided areas connected to the sanitation network and areas that frequently flooded. Arrondissements are subdivided into 43 communes d'arrondissement or CAs (admin3 and 4 on the map). The majority of households surveyed are located in five arrondissements: Pikine Dagoudane (center-west, 14%), Thiaroye (center, 30%), Guédiawaye (center-east, 14%), Niayes (north-east, 37%), and Rufisque (south-east, 5%). Note that households in Rufisque were not sampled in the second and third wave of the surveys, which explains the smaller number of observations (250 unique households).

hosts nearly 80 trucks, while some informal garages host only a handful. There is also a group of independent truckers who operate outside of the garage system and are typically contacted by clients on the street or by cellphone. Our survey provides limited coverage of these truckers as they can be difficult to locate, but discussions with market participants revealed that they tend to operate older and less fuel-efficient trucks.

Much of our analysis focuses on peri-urban residential neighborhoods of Dakar, excluding the east-most neighborhoods of Rufisque, where relatively few households are connected to the sewage network and take-up of mechanized desludging is low (44%). In these neighborhoods, trade is influenced by the Association of Desludging Operators (or AAAS), which controls the operation and prices of the larger garages.[8] We estimate that 50% of trucks belong to a company in which at least one truck has ties with AAAS, and 28% of drivers report being directly affiliated. The official role of the Association is to help operators collaborate on the procurement of truck parts and to assign large, lucrative government and commercial contracts. The influence of the Association likely extends beyond member companies and affects the provision of residential contracts throughout the city. This is because AAAS is involved in the largest garages in the city (where non-member trucks also sometimes park), and distributes contracts and services to member and non-member companies. The threat of being excluded represents a risk of reduced profits due to the loss of non-residential contracts and more difficult access to truck parts.

The cost of providing a mechanized desludging includes the time and fuel required to complete the job (while pumping sludge at the client's house and driving from the garage to the client to the treatment center and back to the garage again), the treatment center's dumping fee[9] and, in some cases, a referral commission paid to the garage. Mechanized desludging exhibits economies of scale due to truck maintenance and/or rental costs. The large majority of drivers, 92%, are paid a fixed salary, and about half report paying a commission for jobs the garage refers to them. A portion of these revenues is redistributed to company owners in the form of revenue-sharing agreements. For example, desludgers in the largest garages report being paid by their garage on days when they do not find work.

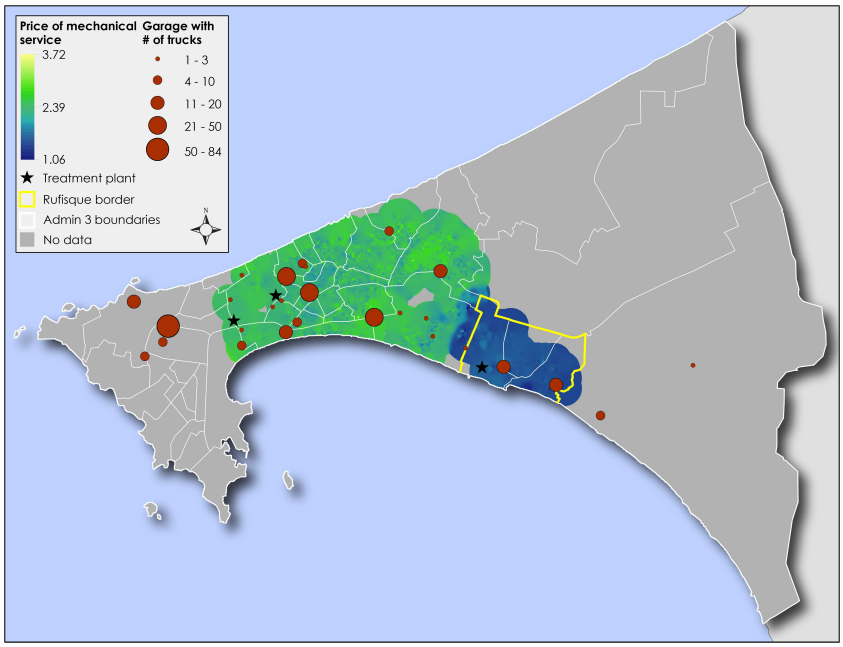
The extent to which firms can benefit from a thicker market or punish one another for deviating from collusive agreements depends on excess capacity in the industry, for which we find ample evidence. To estimate the production capacity of trucks, we ask operators how many desludging trips they performed over the last ten days. On average, trucks perform slightly more than one trip per day (residential and non-residential combined), but there is substantial heterogeneity across trucks. The top ten percent of trucks in terms of number of trips perform more than three jobs per day, and the most active truck performed over 50 trips in a ten-day period. Based on these data

[8] The historical center of the city is wealthier and relatively well connected to the sewer network. The east-most neighborhoods of Rufisque are not controlled by the Association and have a more competitive mechanized desludging market.

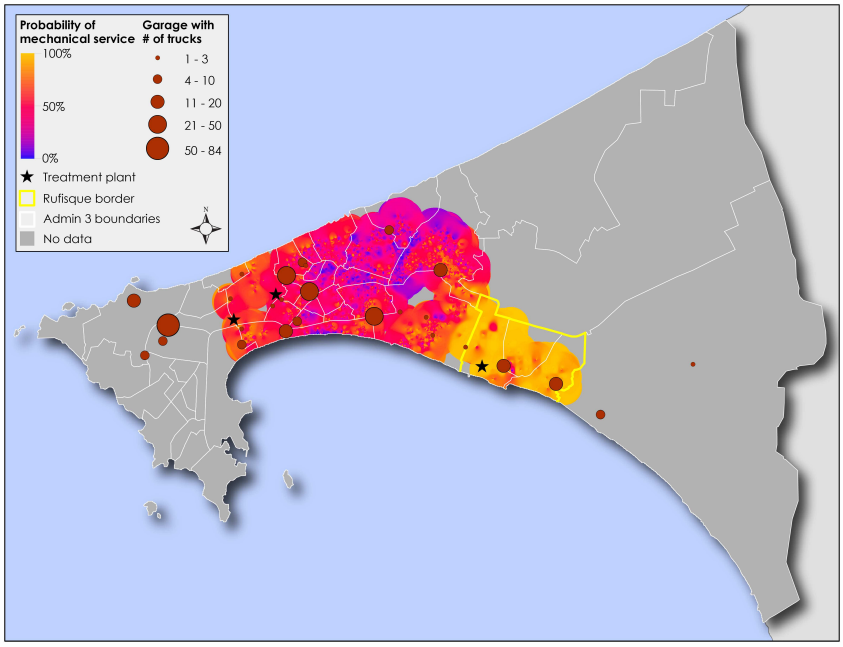
[9] The fee for disposal of the sludge at a treatment center is approximately 3,000 CFA.

Figure 1: Spatial distribution of average prices and demand for mechanized desludging

(a) Average transaction prices



(b) Transaction probabilities



Note: The distance between the Camberene treatment center (the west-most star) and the Rufisque treatment center (the east-most star) is 22 km. Prices are measured in 10,000 CFA.

Table 1: Summary statistics on desludging choices and prices

VARIABLES	(1) Mean	(2) Std. Dev.	(3) Min	(4) Max	(5) N
Mechanical price (x1K)	22.82	7.61	10	50	4865
Baay pell price (x1K)	14.28	6.5	5	40	2339
Family price (x1K)	0.53	2.54	0	25	2601
Choice: Mechanical	0.5	0.5	0	1	9970
Choice: Baay pell	0.24	0.43	0	1	9970
Choice: Family	0.26	0.44	0	1	9970
Truck find: Garage	0.22	0.42	0	1	4865
Truck find: Phone	0.19	0.4	0	1	4865
Truck find: Referral	0.43	0.49	0	1	4865
Truck find: Street	0.1	0.3	0	1	4865
Truck find: Other	0.05	0.23	0	1	4865

Note: Prices and search method, conditional on the desludging type chosen, from the household survey data. Prices are measured in 1,000 CFA. Mechanized prices are censored to be between 10 and 50 thousand CFA while baay pell prices are censored to be between 5 and 40 thousand CFA (the 5th and 99th percentile values).

and discussions with providers, we estimate that a typical job takes about two hours from start to finish. Most truckers operate with substantial excess capacity, while only a small fraction operate at full capacity. Eighty-five percent of desludging operators in the baseline provider survey stated that they could find more jobs if they wanted to make more money.

Since driving the truck and operating the vacuum pump require significant amounts of fuel, the cost of diesel and the efficiency of the truck play an important role in determining trucker costs. The fuel efficiency of the trucks varies substantially with truck size and age, and we estimate that most trucks get between three and six kilometers per liter of diesel. Since diesel prices averaged 750 CFA per liter over the period, the fuel cost per kilometer ranges between 125 CFA and 250 CFA.[10] Conversations with market participants also reveal that a single latrine pit usually fills the truck more than halfway, limiting the ability of drivers to service multiple clients without dumping the sludge at a treatment center in between. About 8% of households require more than one trip.

Table 1 presents summary statistics on transaction price and desludging technology choice from the household survey. Despite the health hazards associated with manual desludging, the reported market share of the mechanized service is only 50%. This is mostly due to the price difference between the two options. Hiring a baay pell to conduct a manual desludging tends to cost between 12,000 and 16,000 CFA ($24-$32), with an average of 14,300 CFA. Most households that have a family member conduct their manual desludging do not pay anything for the service. In contrast,

[10] In order to estimate the amount of diesel necessary per kilometer for a job, we sent an enumerator on ride-alongs with two truck drivers, filling the tank at the beginning and end of the day and recording the kilometers traveled and diesel used.

households pay on average 22,800 CFA per trip (approximately $46) for a mechanized desludging.

Figure 2 shows the distribution of transaction prices from the household survey. Most transaction prices for mechanized desludgings are multiples of 5,000 CFA. Roughly 30% of mechanized desludgings cost 25,000 CFA, with 56% of transactions costing 20,000, 25,000, or 30,000 CFA. We observe similar coarseness in the distribution of prices for manual desludging. While this is mostly due to the fact firms find it more efficient to use round numbers when performing cash-based transactions (Beaman et al. 2014), the fact that a large number of transactions have prices that are not multiples of 5,000 CFA suggests that this does not represent a hard constraint for providers.

3 Non-experimental evidence of collusion

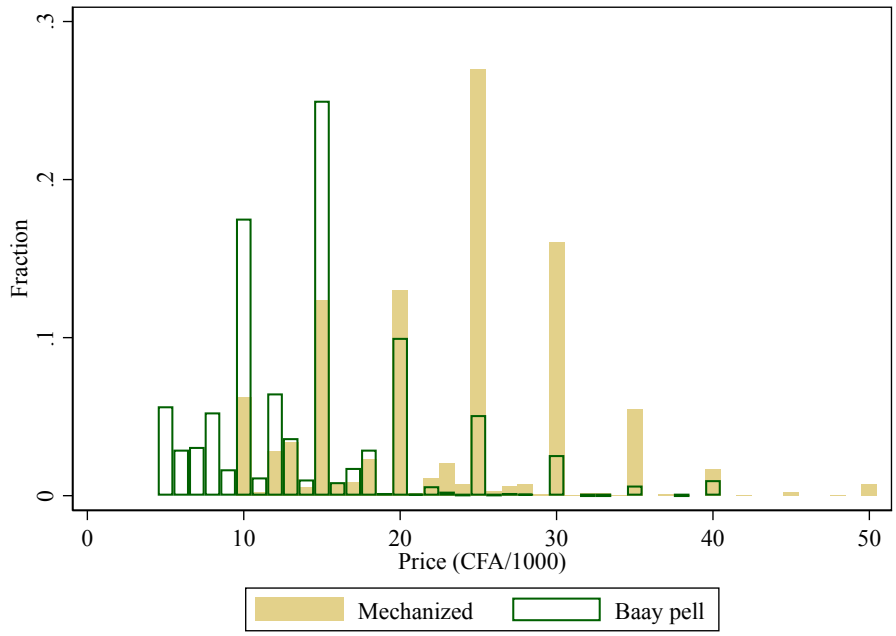
In this section, we provide non-experimental evidence consistent with the presence of collusion in the mechanized desludging market. In particular, we characterize the distribution of prices and demand in the areas of Dakar that are controlled by AAAS. These neighborhoods correspond to areas in which collusion is more likely to be prevalent, as opposed to neighborhoods in Rufisque in which firms operate independently. We use this analysis to provide preliminary evidence on the importance of collusion in the market.

Prices are much lower in the Arrondissement of Rufisque on the eastern outskirts of Dakar which the Association does not control. In the 1990s a single company, UPAMA, provided desludging services in the independent municipality of Rufisque. UPAMA's main line of business was desludging for fish product processing companies, and it was asked by the city administration to provide affordable residential desludging. Over time, new companies entered the market to serve growing demand, but the new companies matched UPAMA's base price in order to get business. The proximity of the Rufisque treatment plant helps reduce the variable cost of the service relative to other areas of Dakar. UPAMA receives no direct subsidies, and we believe the price reflects the cost of providing a mechanized desludging in the area.

Figure 1 plots the (smoothed) distribution of prices and demand across the neighborhoods of Rufisque and the rest of the city. The official boundaries of Rufisque are highlighted in yellow (in southeast Dakar). Panel A shows that the median price for a mechanized desludging in Rufisque is roughly 15,000 CFA, compared to 25,000 CFA in the rest of the city. This means that the price of a mechanized desludging in Rufisque is roughly equivalent to the price of a manual desludging. As a result, as shown in Panel B, nearly all households in Rufisque choose the mechanized option, compared to roughly 40% in the rest of the city.

Appendix Table A-1 formally tests for differences in observable characteristics of households in Rufisque versus the rest of Dakar. Households in Rufisque tend to be closer to both a treatment center and a garage, leading companies operating in Rufisque to have lower variable costs. Home ownership is higher in Rufisque than in the rest of Dakar, reflecting the fact that many people move out to the periphery of Dakar in order to be able to build their own homes.

Figure 2: Distribution of transaction prices for mechanized and manual desludgings



To account for differences between households in Rufisque and other parts of Dakar, we estimate the following regression relating mechanized desludging transaction prices p for household i in month t and household characteristics (including distance to the Rufisque boundary):[11]

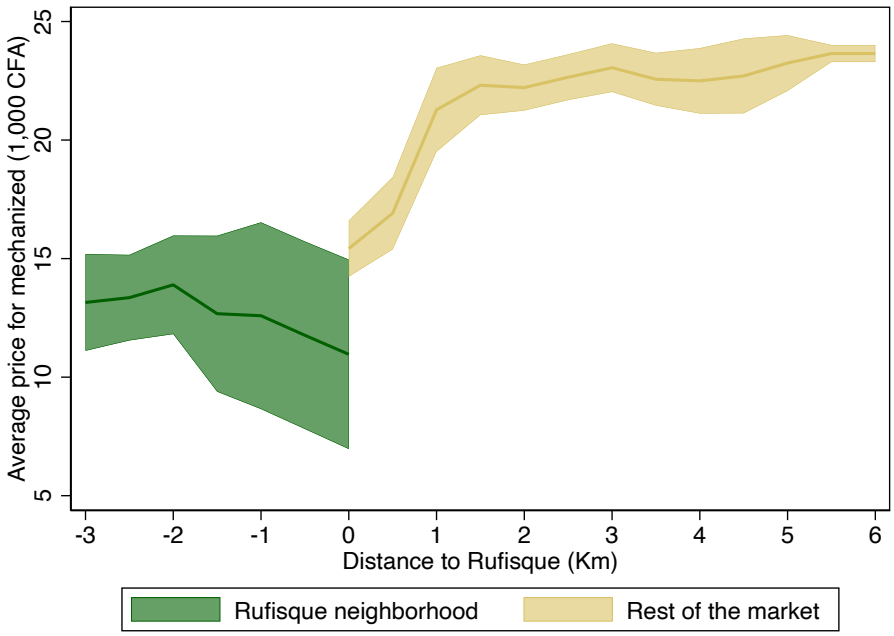
$$p_{it} = g^k\left(\text{Distance to Rufisque}_i\right) + x_{it}\beta^k + \epsilon_{it} \quad k = \text{Rufisque, Other.} \tag{1}$$

We estimate this regression separately for the two regions to allow for differences in pricing strategies, and approximate $g^k(\cdot)$ using a step function of distance with 500 meter increments. In addition to household characteristics, the regressions also control for month-year fixed effects and standard errors are clustered at the household level. Note that since Rufisque is located in the southeast portion of the city, we measure the distance to Rufisque using distance to the nearest western or northern Rufisque boundary. Table A-2 in the Appendix presents the results of this regression for transaction prices.

Figure 3 presents the predicted values from these regressions by distance to the Rufisque boundary. The outcomes are predicted using the average characteristics of households living in Rufisque to eliminate any composition differences. The green line presents the predicted prices in Rufisque,

[11] We control for the following characteristics: distance to the nearest treatment center, distance to the nearest garage, a wide road indicator, household size, number of rooms in the house, a two-story house indicator, a house ownership indicator, a household wealth index, the number of earners living in the same household, and the number of other households living in the same house.

Figure 3: Predicted prices as function of distance to Rufisque



Note: Predicted prices are evaluated at the average household characteristics of households in Rufisque, with 95% confidence intervals.

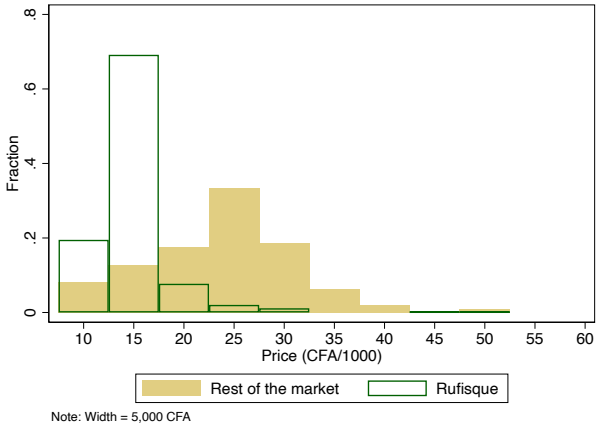
and the orange line presents the predicted prices in the rest of the market. The solid area represents the 95% confidence interval.

This analysis confirms that lower prices in Rufisque are not due to observed differences in household or location characteristics. In the rest of the market, which is controlled by AAAS, mechanized desludging prices increase rapidly with distance from the Rufisque border, but predicted prices are flat with respect to distance in Rufisque. Households in Dakar living within 500 meters of the official Rufisque boundary pay nearly the same price as their neighbors in Rufisque; roughly 15,000 CFA. The gap widens significantly as we move more than 1 km away from the boundary. Low prices in Dakar near the border of Rufisque may be due to competition spilling over the boundary, or may be because the official boundary differs from the de facto boundary. Households in Dakar more than 1.5 km from the Rufisque border pay close to 25,000 CFA, and the price schedule is independent of distance to Rufisque at further distances. In contrast, if one looks at prices for manual desludgings (not shown here), they do not differ significantly across the region, although so few residents of Rufisque get a manual desludging that we do not see many observations of manual prices within Rufisque.

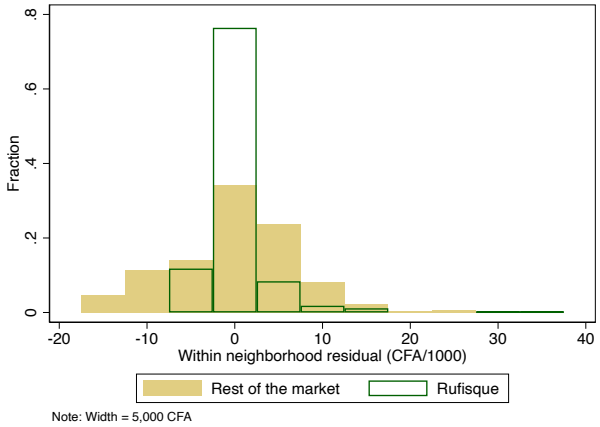
Figure 4 illustrates another important difference between the two areas: price dispersion. Since mechanized desludging prices in Rufisque mostly reflect mechanized desludging costs, we observe

Figure 4: Distribution of mechanized prices in Rufisque and in the rest of the market

(a) Price levels



(b) Price residuals



Note: The price residual is computed as the difference between each transaction price and its zone average (27 zones).

very limited dispersion in transaction prices across households. In contrast, prices are very dispersed in the areas controlled by the Association in the rest of Dakar. Roughly 25% of households in the rest of Dakar pay mechanized prices comparable to those paid in Rufisque. The remaining 75% of households pay higher prices, and a sizable fraction pay more than double the average Rufisque price.

Importantly, Figure 4b shows that prices are dispersed across consumers even within narrowly defined neighborhoods. The figure plots the distribution of price residuals obtained by taking the difference between transaction prices and the corresponding neighborhood averages (27 neighborhoods). The inter-quartile range (IQR) of this residual price is 1,800 CFA in Rufisque, compared to 8,600 CFA in the rest of the market. This suggests that the main component of desludger costs, household location, only explains a small fraction of the observed dispersion in prices paid by consumers in the areas controlled by the Association. This is consistent with the presence of imperfect competition in the market, potentially due to price discrimination and search frictions.

In summary, this analysis suggests that there are differences in competitive conduct between areas that are controlled by the Association and the Rufisque neighborhood. Assuming that unobserved cost differences are continuously distributed around the Rufisque boundary, the results establish that the average household in the Association-controlled neighborhoods pays a significantly higher markup for mechanized desludgings than the average household in Rufisque. This is consistent with the hypothesis that the Association is successful at restricting supply and maintaining high prices in most areas of Dakar. In the next section, we test for collusion using data from the experimental auction platform.

4 Experimental design and evidence of collusion

We now turn to the evidence for collusion that can be inferred from exogenous randomization in the auction platform, rather than exploiting economic and geographic features of Dakar to explore price variation in the traditional market. Collaborating with the government of Senegal, we designed a centralized auction platform to improve access to desludging services by increasing competition between operators. An added benefit was that this allowed us to analyze bidding behavior in a controlled environment. Although consumers endogenously chose to contact the platform in order to request price quotes, the supply side of the auctions was designed to include multiple sources of controlled randomization.[12] In particular, we randomized both the auction format as well as how many and which bidders were invited to each auction. By repeatedly observing desludger participation and bidding behavior under randomly selected auction formats, we can measure firms' propensity to behave competitively. In this section, we start by describing the structure of the auction platform and the sources of random variation. We then describe our identification strategy and show results testing for collusion using auction- and bid-level outcomes. Finally, we measure the effect of collusion on prices paid by consumers. In Section 5, we attempt to rule out explanations other than collusion for the empirical patterns uncovered.

4.1 Description of the auction platform

The design of the platform is simple in order to encourage participation by actors on both sides of the market: households who need the service only sporadically, and desludging operators who are busy and may not have time to engage in or master complicated bidding processes. The auction platform is entirely phone based. When a household needs a desludging service, it calls the center and gives the dispatcher basic information about the location of the pit to be desludged. The call center dispatcher solicits bids via text message, and desludging operators have one hour to respond by text message. The bidder with the lowest price – after accounting for any penalties associated with past service problems – wins the auction.[13] In the case of a tie, the earliest bid at the winning price is awarded the job. The call center dispatcher makes the winning offer to the client, and if the client accepts, the client's phone number is sent to the winning bidder by text message. The client and the operator are then free to make logistical arrangements on their own, and the household pays the desludging operator directly.[14]

[12] Because clients who choose to use the platform are a select subset of the population, we cannot use the experiments to measure the effect of auctions on desludging prices in the open market.

[13] In our analysis we abstract away from penalties. We do this for two reasons. First, penalties were only applied in the first version of the platform and were discontinued in the later auctions. Second, the platform design was such that it was difficult for bidders to know whether a penalty was applied to their competitors. We therefore believe that it is reasonable to assume that firms behave assuming that the lowest bid wins.

[14] Some customers are surveyed by phone after the desludging takes place. The survey asks about the quality of the service and if they were charged the right amount. Desludging operators know that they will be penalized in future auctions if they charge more than the agreed price or provide low quality service.

The final winning bid, but not the identity of the winner, is sent to all desludgers invited to participate in the auction. We reveal the winning price for two reasons. First, the Association and participating companies requested information about which bids won, which might reflect a desire for transparency and fairness, the ability to monitor and maintain cartel discipline, or feedback about which bids were winning in which locations. Second, we wanted the participants to learn which bids were actually competitive, which would be much more difficult without clear and ongoing feedback about the values of winning bids.

The platform randomizes two components of the auctions: (1) Between 5 and 21 of the 126 registered desludgers were randomly selected to compete in each auction.[15] Invited bidders were informed about how many bidders were invited, but not their identities. (2) The auction format was randomized between *sealed-bid* and *revisable-bid* formats. In sealed-bid auctions, the bidders have one hour to submit their bid and receive no information about other bids that have been made until the winning bid is announced at the auction's conclusion. In the revisable-bid format, bidders are given updates about the standing low bid every 15 minutes and again 10 minutes before the auction closes, and are allowed to submit revised bids at any time. In both formats, desludging operators receive reminder messages that bids are still being accepted every 15 minutes and again 10 minutes before the auction closes.

We use this experimental variation in two ways. First, as we discuss below, we exploit theoretical differences in *competitive* bidding strategies across formats to test for competitive conduct. Since firms and consumers are randomly assigned to auction formats, we attribute differences in bidding behavior across formats to differences in firms' strategies. Second, because invitation lists are randomized and anonymous, firms cannot coordinate their behavior within auctions, and are unlikely to face the same set of rivals in future auctions. We use the auction platform as a "laboratory" to gain insights about how firms compete in the traditional market.

We ran 5,331 auctions for mechanized desludging services through the call center in collaboration with the Senegalese Office of Sanitation (ONAS) from July 2013 through April 2017.[16] We drop auctions with winning bids above 60,000 CFA (6 auctions), auctions that did not receive any bids (971 auctions), and auctions performed for subsidized households (671 auctions).[17] We also drop auctions conducted after November 2016 (159 auctions) since the platform was less advertised and received very few calls in the last six months. The final sample (after dropping auctions in

[15] Invitation probabilities were independent of distance to the household. In the first version of the platform invitation probabilities differed across bidders as a piece-wise linear function of the number of valid bids submitted by a trucker in the prior months. This probability was truncated at the bottom and top to ensure that the invitation probability was bounded away from zero, and was less than 50%. In the later auctions, desludger invitations were unconditionally random.

[16] Before the auctions began, the project held multiple training sessions for the truckers to help them understand the auction process and teach them to bid using the SMS message system. During the auction roll out, operators were available to take calls from the truckers at all times if they had trouble placing their bid.

[17] The subsidy intervention was conducted on a small subset of the Dakar population and is described and analyzed in Lipscomb & Schechter (2018) and Deutschmann et al. (2022).

Table 2: Summary statistics from the auction platform

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Frequency: Revisable auctions	4,485	0.502	0.500	0	1
Frequency: New platform (2015-2016)	4,485	0.465	0.499	0	1
Frequency: Zero bids	4,485	0.191	0.393	0	1
Frequency: Accepted bid (at least one bid)	3,365	0.291	0.454	0	1
Number of valid bids (at least one bid)	3,365	2.352	1.382	1	11
Number of invited bidders per auctions	4,485	12.73	2.692	5	21
Number of auction invitations per bidder	126	453.0	305.2	13	1,459
Bidder participation probability	126	0.102	0.130	0	0.524
Number of auction invitations per active bidder	40	758.8	241.1	367	1,459
Active bidder participation	40	0.235	0.129	0.0689	0.524

Active bidders = More than 30 valid bids submitted.

those non-mutually exclusive categories) contains 3,627 auctions with at least one bid. In January 2015, the management of the platform was transferred to ONAS. Penalties were abolished and the invitation rule was modified so that all truckers had the same probability of being invited rather than over-sampling active participants. Also, the number of truckers invited decreased on average from 14 to 11. Although the changes to the invitation rule affected the performance of the platform (by reducing competition), the assignment of bidders to formats and auctions remained random. Slightly more than half of auctions were performed prior to the design change (53%).

Table 2 summarizes details of the auctions and participants. Because households often reject relatively high winning bids, only about 29% of auctions result in a job being completed. In many cases, auctions receive only a few bids, and in some cases households are matched with distant truckers. The table also illustrates the experience and participation rate of bidders. On average, bidders were invited to bid in 453 auctions. The participation rate is fairly low. The probability of submitting a bid is about 10%, which leads to an average of 2.4 valid bids per auction. This low participation rate is due to the fact that a majority of desludgers rarely or ever bid. As we discuss below, 52 bidders submit more than 20 bids during our sample periods and much of our bidder-level analysis focuses on the 40 most active bidders submitting more than 30 bids.

4.2 Hypothesis and identification strategy

We construct an empirical test of competitive bidding in the auction platform, measuring the prevalence of imperfect competition by exploiting the random assignment of bidders to auctions and auction formats. We interpret a rejection of the null hypothesis of competitive bidding as evidence of tacit collusion. A bid is deemed 'competitive' if it is consistent with individual profit

maximization. Conversely, bidders who systematically avoid these behaviors are deemed 'collusive,' since their actions are at odds with individual profit maximization (Chassang et al. 2022*b*, Porter & Zona 1993, 1999). We follow this strategy of testing the null hypothesis of competitive bidding because we do not have a model of collusion in this environment. Economic models of tacit collusion can be used to determine the factors that facilitate price coordination, but are silent regarding the particular collusive strategy that is selected by firms.

We assume that (a) bidders have rational expectations about the distribution of rival bids, (b) bidders observe independent and private signals of the cost of providing the service, and (c) the underlying cost distribution is continuous, smooth, and does not exhibit any mass points. We believe that these assumptions are reasonable in our context. The rational expectation assumption is justified by the fact that bidders are frequently invited to bid and receive information about the winning bids in all invited auctions, whether or not they participated. In addition, when invited to bid in a revisable-bid auction, bidders are informed about the standing low bid at minute 50, which provides useful information on the distribution of the "bid to beat" and the probability of facing sincere competition. Since bidders also compete in the traditional desludging market, they are well informed about the distribution of transaction prices in each of the neighborhoods as a benchmark for the auctions. As discussed in Section 2.2, the marginal cost of desludging is determined by the distance between the garage and the house and the treatment center, the age and size of the truck, and capacity utilization. It is therefore unlikely that the cost distribution exhibits mass points.

As a first source of evidence about collusive behavior, we investigate the presence of ties due to identical lowest bids in the sealed-bid auctions. Excessive correlation in bids is a common red flag used by antitrust authorities.[18] Identical bids can reflect a tacit agreement between firms to use focal prices. Bidding focal prices softens competition by allocating the job to participants who submit early bids. This is a sub-optimal strategy, since the use of focal prices to rotate the identity of the winner creates mass-points in the distribution of winning bids, and a bidder can do strictly better by bidding slightly below these focal prices, increasing their probability of winning the auction substantially for an very small reduction in the price they will be paid.[19]

Our second focus is on the timing of bidding. In particular, bidding in the final ten minute closed portion of the revisable-bid auction is profitable for competitive firms. It allows bidders to jump in at the end and "snipe" the standing low bid by slightly undercutting it (Bajari & Hortaçsu 2003, Roth & Ockenfels 2002). If bidding is costly (in our case the cost of sending a text message), competitive firms are better off learning about rivals' bids before bidding, rather than submitting multiple bids over the course of the auction. In contrast, when trying to collude, firms benefit from bidding early in two ways. First, in the case of collusion, bidding early increases the information

[18] Antitrust laws are not well enforced in Senegal, and the fear of being detected does not play an important role. Mund (1960) and Comanor & Schankerman (1976) provide early analyses of identical bids used by cartels, and McAfee & McMillan (1992) provides a theoretical discussion of the efficiency of this type of strategy.

[19] Section 5 discusses and Appendix C constructs a model of collusion which results in round bidding.

provided to rivals, and therefore reduces the likelihood that the bid will be undercut. Second, for both competitive and collusive firms, in the case of a tie the bidder who bid that amount first wins the job, so that early bidding confers an advantage if a tie occurs.[20]

Consistent with the idea that it is optimal for a competitive bidder to bid in the final ten minutes of the revisable-bid auctions, desludger Cheikh Gueye explains how he learned to bid in his 2018 *Planet Money* Poop Cartel interview (Planet Money 2018): "Generally, what I do is I wait until there are only ten minutes left. If no one takes the offer, then I propose a price. And then immediately, I go so that I have this market." The interviewer then asks: "You said earlier that the truckers were united. Did the text messages – did the auction make you less united because you were competing with each other for price?" And Cheikh Gueye responds, "Even though we used to be united – but now it's a competition. And you need to work hard in order to get something in your business."

Given these hypotheses, we test for imperfect competition by identifying behaviors that fail to maximize expected profit: ties in sealed-bid auctions and early bidding in revisable-bid auctions. Of course, deviation from profit maximization can arise for reasons other than collusion. For example, Hortaçsu et al. (2019) rationalizes the presence of non-serious bids using a level-K model of bounded rationality, assuming that bidders behave non-cooperatively. A related violation of rationality is the presence of menu costs or other pricing frictions that restrict the ability of firms to select new prices or revise their previous bid choice to maximize profits, which could potentially explain the ties at focal prices. Finally, another possibility is that firms submit non-serious bids in order to be invited more often and/or receive other government assistance. These are important caveats that apply to many papers testing for collusion. We discuss the empirical relevance of these alternative interpretations in Section 5.

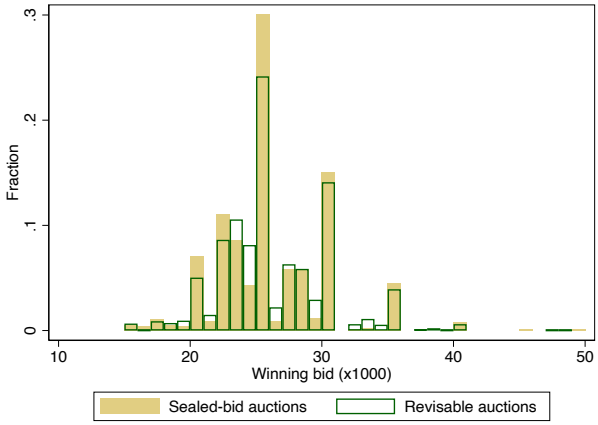
4.3 Empirical analysis

We compare auction-level outcomes across the two formats in order to identify behavior inconsistent with competitive bidding in Section 4.3.1. We then leverage the panel dimension of the data to analyze heterogeneity in competitive conduct at the bidder level in Section 4.3.2. Because we see the same bidders invited to a large number of auctions, we look at whether bidders who systematically choose a sub-optimal strategy in one format also choose a (different) sub-optimal strategy in the other format. Similarly, we identify competitive bidders who have a high propensity to avoid sub-optimal strategies in both formats, adjusting their bidding strategy depending on the auction type. To conduct this analysis, we leverage the fact that bidders are randomly invited to bid in different

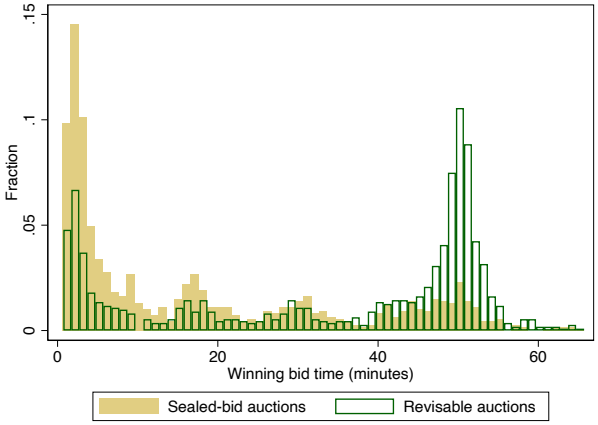
[20] One potential downside of bidding late is that it may increase the probability that a bid is rejected by the platform due to technical delays. In our platform however, bidders have ten minutes to submit a bid after the last message, and the platform gives an additional five minute grace period to ensure that all bids are received. During this period, the auction turns into a first-price sealed-bid auction with a reserve price defined by the lowest bid received prior to minute 50.

Figure 5: Distribution of winning bids and arrival times across the two formats

(a) Distribution of winning bids



(b) Distribution of winning bid time



auctions formats. Finally Section 4.3.3 looks at the effect of collusion on prices in the auction platform, which is especially important given the sensitivity of consumers to price and the negative health effects of manual desludgings. Table 3 presents summary statistics of the outcome variables at both the auction and bid levels.[21]

4.3.1 Auction-level analysis

We start by analyzing the distribution of winning bids and their arrival times for revisable-bid and sealed-bid auctions. Figure 5a plots the histogram of winning bid amounts. The distribution in the sealed-bid format exhibits clear mass points at common focal prices: 20, 25, 30, and 35 thousand CFA. The distribution of winning bids in the auctions closely resembles the distribution of negotiated prices in the traditional market in areas of Dakar outside of Rufisque displayed in Figure 4. The same mass points are present in the revisable-bid format sample, but there are clear differences. Winning bidders in the revisable-bid format are more likely to undercut those focal prices by 1,000 or 2,000 CFA, which leads to a higher density at 23, 24 and 29 thousand CFA. This implies that cash-transaction frictions cannot fully explain the use of focal prices in both the traditional market and in the sealed-bid auctions.

Figure 5b shows that in sealed-bid auctions, roughly 30% of winning bids are placed in the first 5 minutes. Since the tie-breaking rule favors early bidders, bidding early is an optimal strategy for all bidders irrespective of their propensity to collude. In the revisable-bid auctions, the share of early winning bids is much smaller, and the modal winning bid is placed after the last message (i.e., after

[21] Note that the minimum bid received is lower than the winning bid. This is because in 17 auctions the lowest bidder is not the winner due the presence of penalties. Since this occurs rarely, we abstract away from penalties in this paper.

Table 3: Summary statistics of auction-level and bid-level outcome variables

(a) Auction-level variables

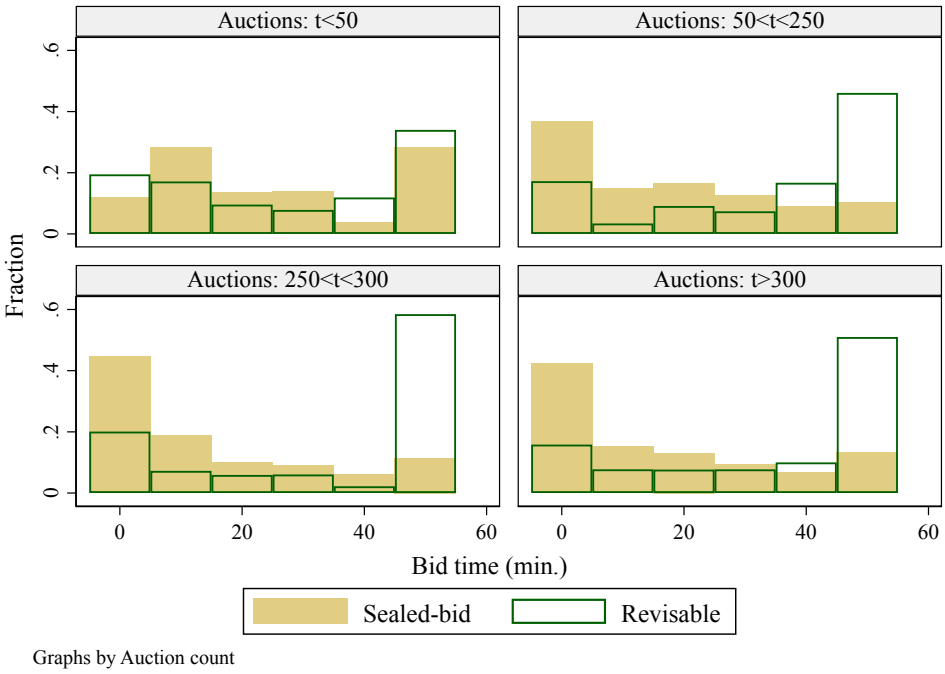
VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Winning bid amount (x1,000 CFA)	3,627	25.73	4.140	15	50
Winning bid time (min.)	3,627	29.85	21.57	0.650	65.20
1(Winning ties — Auction)	2,503	0.191	0.393	0	1
First bid amount (x1,000 CFA)	3,627	27.67	4.664	15	50
First bid time (min.)	3,627	17.17	19.10	0.650	65.20
1(Round winning bid)	3,627	0.526	0.499	0	1
1(Accept)	3,627	0.296	0.457	0	1

(b) Bid-level variables

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
1(Bidder participation)	26,279	0.300	0.458	0	1
Bid amount (x1,000 CFA)	8,918	27.14	4.524	14	50
1(Round bid)	8,918	0.608	0.488	0	1
1(Late bid)	8,918	0.205	0.404	0	1
Bid time (min.)	8,918	26.69	20.62	0.650	65.93
1(Tie bid — sealed-bid)	3,510	0.366	0.482	0	1
1(Undercut — revisable)	1,741	0.819	0.385	0	1

Note: Panel A shows the amount and time of the winning bid, an indicator for the winning bid being a tie (conditional on there being at least two bids), the amount and time of the first bid, an indicator for whether the winning bid is a multiple of 5,000, and an indicator for whether the client accepted the desludging to be conducted at the winning bid. Panel B shows the probability that invited active bidders submit a bid. Conditional on submitting a bid it shows the bid amount, an indicator for whether the bid is a multiple of 5,000, an indicator for whether the bid was submitted in the last ten minutes, and the time of the bid for each bid. Conditional on there being at least two bids in an auction, it shows an indicator for the bid being a tie in a sealed-bid auction. Conditional on there being a bid in the price information message, and conditional on the bid being after the price information message, it shows an indicator for being lower than the price information message in a revisable-bid auction. Panel B uses the last bid submitted by bidders who submitted more than one bid in the revisable-bid format. Bids larger than the 99.5th percentile are winsorized.

Figure 6: Distribution of winning bid times across auction



minute 50). Between these two extremes, the distribution of bid times reflects the nudges created by the messages. The difference in behavior across the two formats is consistent with the idea that it is optimal for competitive bidders to submit early bids in sealed-bid auctions and late bids in revisable-bid auctions. However, the fact that a significant fraction of winning bids in revisable-bid auctions arrive early suggests that not every bidder behaves competitively.

Interestingly, Figure 6 shows that bidders learned fairly quickly how to adjust their strategies. In the first 50 auctions, the bid times were very similar in the two auction formats. The two distributions diverged over time. The winning bid times in sealed-bid auctions converged to a distribution with declining density, while the winning bid times in revisable-bid auctions converged to a bimodal distribution with significant mass both very early and very late. In the revisable-bid format, late bidding was relatively rare in the early auctions, but became the modal outcome starting around the 200th auction. The share of early winning bids fluctuates somewhat before eventually stabilizing at around 18%. Many successful winning bidders learned that it is good to bid competitively in the last ten minutes of the revisable-bid auction, while a significant share of winning bids were placed surprisingly early. Learning also took place in the sealed-bid format. The fraction of early bids increased over time, from about 30% in the first 50 auctions to more than 50% after the 300th auction.

Since the format is randomly assigned to each job, we can use a simple auction-level treat-

ment effect regression to summarize the difference in outcomes across revisable-bid and sealed-bid auctions. We estimate

$$y_t = \alpha \, \text{Revisable}_t + x_t\beta + \epsilon_t \tag{2}$$

where y_t measures one of five different outcomes for auction t: (i) the winning bid amount, (ii) an indicator for the winning bid being tied, (iii) an indicator for the winning bid being divisible by 5,000, (iv) an indicator for the winning bid occurring in the last time interval, and (v) the minute of the first bid. Each regression controls for auction and consumer characteristics to increase efficiency, but the results are unaffected by their inclusion. Appendix Table A-3 presents the mean and standard deviations for the main outcome and control variables used in the analysis. Note that the distance variables are omitted from the auction-level regressions as they vary across bidders.

Table 4 reports the results, while Appendix Table A-4 omits the intercept from the regression and presents the conditional means in both formats. The first column tests whether the winning bid differs across the two formats. The point estimates suggest that revisable-bid auctions lead to winning bids that are 100 CFA larger than sealed-bid auctions, but the difference is not statistically significant.[22] Although it is interesting to learn about the effect of the format on average winning bids, from a theoretical perspective there is no reason to believe that the two formats should be revenue equivalent (under either collusion or competition). This is because the revisable-bid auction format has a "hard close" and bids submitted in the last 10 minutes are not observed by rivals. The revisable-bid auction is best described as a sequential auction: open followed by closed.

The next two columns analyze the prevalence of ties and round bids. The probability that the winning bidder ties is 9 percentage points higher in the sealed-bid auction. As column (3) illustrates, this is explained by the fact that firms are significantly more likely to use bids which are divisible by 5,000 in the sealed-bid format. This is consistent with Figure 5a above. By revealing the current lowest bid at the 50th minute, the revisable-bid auction allows competitive bidders to undercut the standing low bid as of minute 50, and win the auction more often. Note, however, that the fraction of round bids and ties does not go to zero in the revisable-bid format. On average, 14.4% of revisable-bid auctions end in a tie, compared to 23.6% of sealed-bid auctions.

Columns (4) and (5) analyze the timing of bids. As Figure 5b suggests, the winning bid is 28 percentage points more likely to be placed in the last ten minutes of the auction in the revisable-bid format. Similarly the first bid is received three minutes later in the revisable-bid format (minute 19 versus minute 16). The fact that the first bid arrives relatively quickly in both formats explains the bimodal distribution of winning bid times. Most bidders in the sealed-bid auctions submit bids immediately after receiving the invitation, while a large fraction of winning bidders in the revisable-bid format submit late bids (roughly 42%).

[22] In general, collusion is thought to be easier to sustain in open auction environments (without a hard close). See Robinson (1985), Graham & Marshall (1987), Marshall & Marx (2007), and Athey et al. (2011) for theoretical and empirical analyses in the context of English auctions. Our context differs from a standard English auction in that there is a time limit and we do not reveal all bids to the bidders.

Table 4: Experimental treatment effect of auction format on bidding strategies

VARIABLES	(1) Winning bid	(2) 1(Ties)	(3) 1(Round)	(4) 1(Last message)	(5) First bid (min.)
1(Revisable)	0.101	-0.0884^a	-0.0927^a	0.281^a	3.216^a
	(0.112)	(0.0156)	(0.0162)	(0.0141)	(0.571)
Observations	3,627	2,503	3,627	3,627	3,627
R-squared	0.353	0.054	0.082	0.145	0.220
Unit of observation	Auctions	Auctions	Auctions	Auctions	Auctions
Mean dep. variable	25.73	0.191	0.526	0.279	17.17

Note: The dependent variables are (1) the value of the winning bid, (2) an indicator for the winning bid being tied, (3) an indicator for the winning bid being a multiple of 5,000, (4) an indicator for the winning bid coming in the last ten minutes, and (5) the minute the first bid came in. The sample includes auctions with at least one valid bid in all columns, and with at least two valid bids in column (2). Additional control variables include: number of invited bidders (log), distance from client to nearest treatment center, average distance of invited bidders' garages to client, client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Robust standard errors in parentheses. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$.

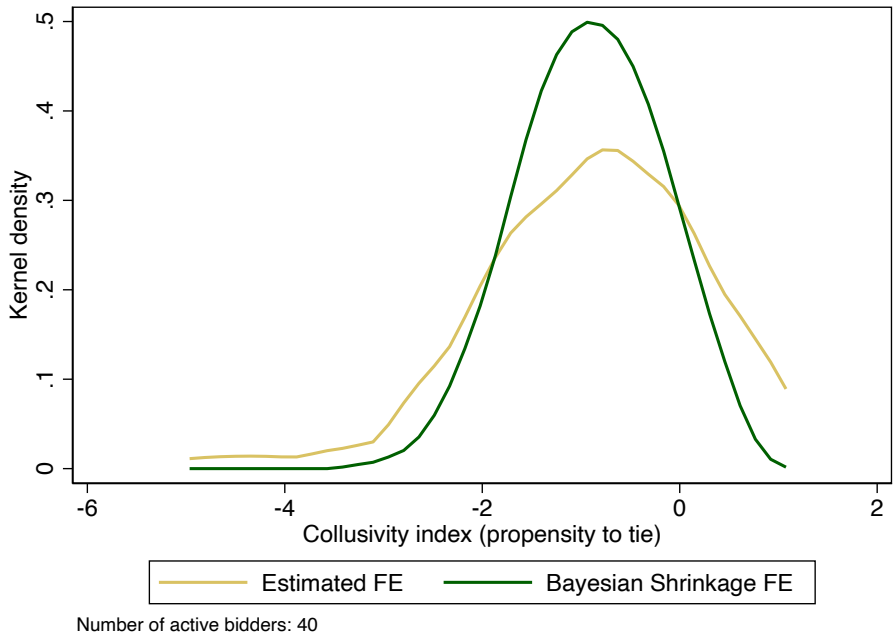
This analysis confirms the presence of sub-optimal behavior in the two formats; most notably the prevalence of ties in sealed-bid auctions, and the fact that a large fraction of winning bids are received early instead of late in the revisable-bid auctions. If all bidders were behaving competitively by maximizing expected profits, bids in the revisable-bid auction would converge to a single mass after the 50th minute, and ties would be very infrequent in both formats.

4.3.2 Bidder-level analysis

The previous section highlighted the presence of non-competitive behavior, while also highlighting the fact that many winning bids appear to be competitive. For example, the distribution of winning bids shows that a nontrivial fraction of bids are placed using a fine price grid. Similarly, the bimodal distribution of winning times in the revisable-bid auctions reveals a mixture of early and late bids. This heterogeneity could in principle be caused by differences in bidding strategies across auctions, with bidders submitting competitive bids for certain types of clients or time periods, and submitting insincere bids for others. Alternatively, this heterogeneity could be due to systematic differences across bidders. In this section, we show that the results are consistent with the existence of a group of competitive/sophisticated firms (last-minute bidders, finer price grid), bidding against a group of non-competitive firms (early bidders, focal prices).

To measure the importance of heterogeneity in bidders' propensity to bid competitively (as opposed to heterogeneity across bids), we next investigate the correlation in bidders' strategies across formats. Specifically, we test the hypothesis that certain bidders behave competitively in

Figure 7: Distribution of the collusivity index across active bidders



both formats, while others choose sub-optimal strategies in both formats. To do this, we first construct a 'collusivity index' for each bidder which is an estimate of the probability that the bidder ties in sealed-bid auctions. A higher value of the index means a lower propensity of bidding competitively. We run the following probit model:

$$\Pr(\text{Tie}_{it}|x_{it}, \theta_i) = \Phi(-x_{it}\beta - \theta_i) \tag{3}$$

where θ_i is bidder i's fixed effect and it measures the bidder's propensity to tie. Low θ bidders are less likely to tie, and more likely to bid competitively. The control variables include the same ones listed in Table 4, in addition to the distance between the client and the driver's garage (measured by a series of dummies for each one km of distance).

To reduce the importance of measurement error in $\hat{\theta}_i$, we focus on active bidders submitting bids in at least 30 sealed-bid auctions. Since those bidders participate at a much higher rate than the average (30% compared to 10%), this sample includes the most experienced and attentive bidders.[23] There are 40 bidders who satisfy this criterion (out of 96 bidders who submitted at least one bid). We estimate equation (3) using the sample of auctions with at least two valid bids.

Figure 7 illustrates the distribution of the collusivity index across active bidders. The index is scaled in standard-normal units, since we estimate the probability of ties using a probit model.

[23]The results are robust to varying the activity threshold between 20 and 40, as well as estimating the fixed effects using a linear-probability model instead of probit.

On average, the probability of ties is 20%. Bidders with a collusivity index above zero have a likelihood of tying over 50%, while bidders with an index below -2 almost never tie. We also create a collusivity index using a Bayesian shrinkage correction following the approach discussed in Chandra et al. (2016). This attenuates the importance of measurement error.[24] Of the 40 active bidders, 11 bidders are more competitive having a propensity to tie that is less than or equal to 5% (based on the raw fixed-effect estimates). The majority of bidders are less competitive, and have average predicted probability to tie of 35%.[25]

To analyze the correlation between the bidders' type and aspects of the bidding strategy, we estimate the following bid-level OLS regression:

$$y_{it} = \alpha\hat{\theta}_i + x_{it}\beta + \epsilon_{it} \tag{4}$$

where x_{it} is a set of control variables describing the auction and the client. Standard errors are clustered at the bidder level. The parameter α measures the correlation between the bidder's collusivity index ($\hat{\theta}$) and the choice variable y. We consider six bidder choices as outcome variables: (i) the bid amount in the sealed-bid auctions, (ii) an indicator for a bid being divisible by 5,000 CFA in the sealed-bid auctions, (iii) an indicator for the bid being placed in the last ten minutes, (iv) the time of bid, and (v) an indicator for a bid lower than the last price information message in the revisable-bid auctions.

Table 5 presents the main regression results with the shrinkage correction while Appendix Table A-5 presents the results without the shrinkage correction. Column (1) of panel (a) shows that bidders with a high propensity to tie in the sealed-bid auctions also submit significantly higher bids in those auctions. The difference between the most competitive types (around $\theta = -2$) and most collusive types (around $\theta = 0$) is 4,500 CFA, or about 16% of the average bid placed. The second column confirms that bidders who are more likely to tie are also more likely to submit bids in 5,000 CFA increments. However, all bidder types are equally likely to submit a late bid in the sealed-bid auction (columns (3) and (4)). This is consistent with the idea that both collusive and competitive types have an incentive to bid early in the sealed-bid auctions due to the tie-breaking rule.

Panel (b) shows results for the revisable-bid auction sample. This panel analyzes the probability of submitting a bid that undercuts the price information message and the probability of late bids. Bidders who have a high propensity to tie in sealed-bid auctions also choose non-competitive strategies in revisable-bid auctions. Conditional on submitting a bid, those bidders are 11 p.p. less likely to undercut. Since those bids were placed *knowing* the value of the 'bid to beat,' this shows that bidders who behave sub-optimally in the sealed-bid auction are also more likely to avoid

[24] To implement this correction, we project the estimated fixed effects on observed bidder characteristics: garage fixed effects, number of trucks, and truck size.

[25] Below we define 'competitive' types as having an index below the 30th percentile of the distribution of the collusivity index θ_i.

Table 5: Relationship between bidders' collusivity index and bidding strategies

(a) Sealed-bid auctions

VARIABLES	(1) Bid amount	(2) 1(Round bid)	(3) 1(Late bid)	(4) Bid time
Collusivity index	2.28^a	0.34^a	0.015	2.55
	(0.45)	(0.037)	(0.059)	(4.53)
Observations	3,992	3,992	3,992	3,992
R-squared	0.264	0.189	0.054	0.102
Unit of observation	Bids	Bids	Bids	Bids
Mean variable	27.1	0.66	0.16	24.2
Nb cluster	40	40	40	40

(b) Revisable-bid auctions

VARIABLES	(1) 1(Undercut)	(2) 1(Late bid)	(3) Bid time
Collusivity index	-0.11^b	-0.37^a	-15.0^a
	(0.051)	(0.078)	(2.88)
Observations	1,741	3,490	3,490
R-squared	0.077	0.211	0.199
Unit of observation	Bids	Bids	Bids
Mean variable	0.82	0.30	30.9
Nb cluster	40	40	40

Note: The dependent variables are (1a) the value of the bid, (2a) an indicator for the bid being a multiple of 5,000, (3a) an indicator for the bid coming in the last ten minutes, (4a) the minute of the bid, (1b) an indicator for submitting a bid lower than the price information message, (2b) an indicator for the bid coming in the last ten minutes, and (3b) the minute of the bid. Column (1) of panel (b) is limited to bids placed after a price information message in auctions with more than one bid. The collusivity index for the bidder is created from their probability of tying in sealed-bid auctions, and takes into account the shrinkage correction. Additional controls include: number of invited bidders (log), distance from client to nearest treatment center, distance from garage to client (1 km bins), client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Standard errors clustered at the bidder level in parentheses. *c*: $p < 0.10$, *b*: $p < 0.05$, *a*: $p < 0.01$.

Table 6: Differences in the timing of bids across formats and bidder types

VARIABLES	(1) 1(Late bid)	(2) Bid time	(3) 1(Late bid)	(4) Bid time
Collusivity index x 1(Revisable)	-0.15^a	-6.59^a	-0.37^a	-16.5^a
	(0.042)	(1.94)	(0.11)	(4.65)
Observations	7,482	7,482	7,482	7,482
R-squared	0.336	0.367	0.341	0.371
Unit of observation	Bids	Bids	Bids	Bids
Bayesian shrinkage correction	no	no	yes	yes
Bidder FE	yes	yes	yes	yes
Mean variable	0.22	27.3	0.22	27.3
Nb cluster	40	40	40	40

Note: The collusivity index for the bidder is created from their probability of tying in sealed-bid auctions. Additional controls include: number of invited bidders (log), distance from client to nearest treatment center, distance from garage to client (1 km bins), client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week and bidder fixed effects. All controls are interacted with the Revisable indicator variable to facilitate the comparison with Table 5. Standard errors clustered at the bidder level in parentheses. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$

competition in the revisable-bid auction format.[26]

Columns (2) and (3) show that the correlation between tying in sealed-bid auctions and bidding late in revisable-bid auctions is negative. This is in contrast to the results in panel (a) showing that the collusive types who tie in sealed-bid auctions are no more or less likely to submit late bids in those auctions. In other words, competitive types bid late in the revisable-bid auctions, but not in the sealed-bid auctions. The first bid placed by non-competitive bidders also arrives much earlier in revisable-bid auctions, but not in sealed-bid auctions. The difference between competitive types with $\hat{\theta} = -2$ and colluisive types with $\hat{\theta} = 0$ in revisable-bid auctions is 30 minutes. This is in line with the idea that non-competitive types use their first bid in revisable-bid auctions as a signal to other collusive bidders invited to the auction.

Table 6 estimates the difference in the timing of bids in the two formats across bidders with different values of the collusivity index. In particular, we pool observations across the two formats, and estimate difference-in-differences regressions that exploit the randomness in bidders' invitations (controlling for bidder fixed effects). We also allow all control variables to have different coefficients

[26]There are two reasons for collusive bidders to submit a losing, or complementary, bid in the revisable-bid auction. First, auctions conducted before 2015 included a penalty for poor past service. About 5% of bids submitted during this period had a penalty, implying that the probability of winning by matching the lowest bid was small but positive. Competitive bidders should avoid tying since bidders were not informed about the specific presence of a penalty and the probability of a penalty in the population of bidders. Second, invitation probabilities were increasing in bidders' past participation. This gave active bidders an incentive to submit complementary bids.

across the two formats, in order to replicate the cross-sectional results. The point estimates therefore correspond to the difference between the estimates in panels (b) and (a) of Table 5. The results confirm that bidders who are more likely to tie in the sealed-bid auctions are also significantly less likely to submit a late bid in revisable-bid auctions.

These results confirm that the presence of sub-optimal behavior is driven by heterogeneity in the propensity of bidders to behave competitively. We establish two patterns. First, a set of bidders deviate from the competitive model in different ways in each auction format - tying in sealed-bid auctions and bidding early in revisable-bid auctions. Second, a group of competitive bidders has a higher propensity to choose optimal strategies. This group quickly learned how to use the platform to win clients and offered lower prices.

4.3.3 Effect of collusion on prices on the platform

To evaluate the effect of non-competitive bidding on market outcomes in the auctions, we estimate the effect of inviting more competitive bidders to an auction on the winning bid. We use OLS to estimate the following regression:

$$\text{Winning bid}_t = \text{Market structure}_t \alpha + x_t \beta + \epsilon_t$$

The market-structure variables are meant to proxy for the competitiveness of each auction t, as measured by the number and characteristics of invited bidders. We include three types of market-structure variables. First, we measure the number of competitive types invited by discretizing $\hat{\theta}$ (which is measured for the most active bidders) into five groups, and calculating the fraction of active bidders invited in each category.[27] The first group, which we label as "very competitive" have a collusivity index below or equal to the first decile, and the "very collusive" group have an index above the 90th percentile. Since these shares sum to one, in each specification the median category is omitted, and so the coefficients are expressed relative to bidders with intermediate values of the tie probability fixed-effect. Alternatively, we include the average value of $\hat{\theta}$ among invited bidders. The second group of variables measures the minimum distance among all invited bidders to the client (based on the garage location) and the minimum distance among all invited active bidders. The third set of variables measures the total number of invited bidders, the total number of active bidders, and the total number of active bidders located within 10 km of the client. Since we randomly chose desludgers to invite to bid in each auction, the estimate of α measures the causal effect of distance and competition on the winning bid.

Table 7 presents the results of this analysis. Inviting competitive bidders leads to an economically large reduction in prices. The marginal effect is also monotonically increasing in the collusivity

[27] We define the categories based on the distribution of collusivity index θ_i: (i) very competitive ($\theta_i \leq \bar{\theta}_{0.1}$), (ii) competitive ($\bar{\theta}_{0.1} < \theta_i \leq \bar{\theta}_{0.3}$), (iii) median bidder ($\bar{\theta}_{0.3} < \theta_i \leq \bar{\theta}_{0.7}$), (iv) collusive ($\bar{\theta}_{0.7} < \theta_i \leq \bar{\theta}_{0.9}$), and (v) very collusive ($\bar{\theta}_{0.9} < \theta_i$) where $\bar{\theta}_q$ denotes the qth quantile of the empirical distribution of θ_i.

Table 7: Effect of competition and collusion on winning bids

VARIABLES	(1)	(2)	(3)	(4)
Avg. collusivity index		0.96^a		1.67^a
		(0.17)		(0.38)
V. Competitive (%)	-3.73^a		-1.91^a	
	(0.64)		(0.64)	
Competitive (%)	-0.51		-1.19^a	
	(0.43)		(0.43)	
Collusive (%)	-0.0060		0.20	
	(0.49)		(0.48)	
V. Collusive (%)	0.99		0.18	
	(0.66)		(0.60)	
Min. distance (km)	0.053	0.053	0.060	0.058
	(0.048)	(0.048)	(0.048)	(0.048)
Min. distance (active)	0.10^a	0.10^a	0.091^a	0.093^a
	(0.030)	(0.030)	(0.030)	(0.030)
Nb. Bidders	-0.075^b	-0.076^b	-0.077^b	-0.075^b
	(0.035)	(0.035)	(0.035)	(0.035)
Nb. Active bidders (< 10 km)	-0.23^a	-0.22^a	-0.19^a	-0.19^a
	(0.045)	(0.045)	(0.045)	(0.045)
Nb. Active bidders	-0.035	-0.038	-0.049	-0.049
	(0.039)	(0.040)	(0.040)	(0.040)
Observations	3,626	3,626	3,626	3,626
R-squared	0.321	0.320	0.316	0.317
Unit of observation	Auctions	Auctions	Auctions	Auctions
Bayesian shrinkage correction	no	no	yes	yes
Mean dependent variable	25.7	25.7	25.7	25.7

Note: The dependent variable in each specification is the winning bid (in 1,000 CFA). The collusion/competition categories are only available for active bidders and are defined as (i) very competitive ($\theta_i \leq \bar{\theta}_{0.1}$), (ii) competitive ($\bar{\theta}_{0.1} < \theta_i \leq \bar{\theta}_{0.3}$), (iii) median bidder ($\bar{\theta}_{0.3} < \theta_i \leq \bar{\theta}_{0.7}$), (iv) collusive ($\bar{\theta}_{0.7} < \theta_i \leq \bar{\theta}_{0.9}$), and (v) very collusive ($\bar{\theta}_{0.9} < \theta_i$) where $\bar{\theta}_q$ denotes the qth quantile of the empirical distribution of θ_i. The median group is omitted from the regression. Additional control variables include: number of invited bidders (log), distance from client to nearest treatment center, average distance from invited bidders' garages to client, client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Robust standard errors in parentheses. *c*: $p < 0.10$, *b*: $p < 0.05$, *a*: $p < 0.01$.

index (measured by the tie probability). The marginal effect of inviting a higher fraction of 'collusive' and 'very collusive' bidders are not statistically different from the marginal effect of inviting bidders in the middle of the distribution. We therefore label as 'competitive' those bidders with an index below the 30th percentile (i.e. 11 out of 40 active bidders). The results are qualitatively

similar with or without the Bayesian shrinkage adjustment (i.e. columns 1-2 vs 3-4).

We can evaluate the effect of changing the composition of invited bidders on the winning bid. On average each auction includes 7.13 active bidders, and thus moving one active bidder from the reference group to one of the other categories is equivalent to changing the group share by 14%. Using the estimates from column (3), adding a bidder to the 'very competitive' group would lead to a 268 CFA reduction in prices. Increasing the fraction of 'competitive' bidders by 14% would decrease the price by 166 CFA. Similarly, using column (4), holding fixed the number and distance of invited bidders, a one standard deviation (or 0.18) increase in the average collusivity index leads to a 306 CFA increase in prices. To put these numbers in perspective, note that in column (4) a one standard deviation (3.26 km) increase in the distance to the nearest active potential bidder is associated with a 302 CFA increase in price. Therefore, the magnitude of the effect of reducing the collusivity index within the auctions by one standard deviation is similar to the effect of reducing providers' marginal cost by one standard deviation.

The last five rows report the effect of distance and number of invitees on auction outcomes. Inviting active bidders whose garages are located closer to consumers, and inviting more of them, leads to significantly lower winning bids; more so than inviting a large overall number of bidders. This is because many non-active bidders are unlikely to participate.

In summary, these results show that randomly inviting more competitive bidders to an auction leads to significantly lower winning bids. Based on the effect of the composition of bidders on the final price, we estimate that roughly 30% of active bidders belong to the group of competitive bidders, while 70% are classified as collusive. Appendix Table A-6 shows that increasing the fraction of competitive bidders also leads to higher acceptance rates, but these estimates are much less precise. Changing the degree of competition in the market therefore has ramifications for the adoption of more sanitary mechanized desludgings.

5 Interpretation of the results

We interpret the results in the previous section as confirming the existence of two types of bidders - competitive and collusive. There are, however, a variety of alternative explanations for sub-optimal behavior besides tacit collusion. Among the leading alternatives, it is possible that bidders differ in their degree of sophistication or attention, or that frictions in the market limit the ability of firms to submit bids optimally. Although we cannot rule out these alternative interpretations, we believe that they cannot fully explain the results.

First, there may be logistical reasons for some of the apparently sub-optimal bidding behavior. For example, the fact that transactions are cash based certainly limits the ability of bidders to use a very fine price grid. However, we observe a non-trivial fraction of bids that deviate from the most common focal prices and reduce the probability of ties, for example using 1,000 CFA increments instead of 5,000. This is true both in the auctions and in the prices that consumers

report paying in the traditional market. Similarly, it is possible that bidders submit insincere or losing bids in order to increase the likelihood of being invited to future auctions. This is unlikely to be an important margin since the average participation probability is 15%, and the most active bidder on the platform participated in 60% of auctions in which he was invited to bid.

Second, the presence of boundedly rational agents is certainly an important concern. Even bidders that we classify as competitive appear to make frequent 'mistakes,' sometimes submitting round bids or bidding early in revisable-bid auctions. We also cannot rule out the possibility that some bidders have biased beliefs about the distribution of bids of their rivals, for example, due to inattention. However, this concern is alleviated by the fact that our bidder-level analysis focuses only on the most active bidders. These firms received frequent invitations to bid (multiple times per week on average), and submit a bid in 30% of the cases. As a result, these bidders had ample opportunities to learn the distribution of winning bids. It is therefore unlikely that the bidders we classify as collusive just ignored the fact that using a coarse price grid would lead to frequent ties.

Perhaps most importantly, the revisable-bid auction format gave bidders a real-time measure of the bid to beat, which should substantially simplify the choice of optimal bid to place. Indeed, bidders with a lower propensity to tie in sealed-bid auctions are significantly more likely to submit a late bid, and undercut the lowest standing bid in revisable-bid auctions. There is, however, a large fraction of active bidders who rarely submit late bids, and who revise their bids down in order to increase their likelihood of winning.

Round bidding as optimal collusion. Round bidding is a sub-optimal strategy for an individual firm, but can it be profitable or optimal for the cartel? Collusion in the presence of private information is difficult to analyze since it adds private or imperfect public monitoring to the analysis of what is already a relatively complex Bayesian game. We construct a model in Appendix C which shows that round bidding is an optimal solution even in the static cartel profit-maximization problem. It also shows how round bidding aids monitoring and enforcement of a dynamic collusive agreement.

The intuition comes in two steps. First, it is profit-maximizing for the cartel to restrict bidding and soften price competition to boost the expected payoffs to winning firms, even for a single auction. This occurs because the cartel is maximizing the expected profits of its members, while the platform is roughly trying to minimize the same. The standard solution for the platform of selecting the lowest bid to provide the service creates full separation and minimizes procurement costs. If separation minimizes cartel profits, it follows that pooling equilibria can raise profits by reducing competition on the margin and constraining cartel members from competing away their informational rents. This is round bidding.

Second, the discreteness of round bidding makes it possible to monitor and enforce an agreement. Unlike a fully separating strategy where all histories occur with probability zero and maintaining cartel discipline becomes a complex challenge, deviations from a round-bidding strategy are detected

immediately and can be punished through a price war with temporary or permanent reversion to the fully separating Bayesian Nash equilibrium of the game. If the cartel members aren't sufficiently patient to implement the statically optimal agreement, we show how the cartel selects round-bidding strategies with more bids to compensate low-cost types for failing to undercut higher bids by rewarding them with a higher probability of winning with their prescribed bid. Thus, the more impatient the cartel members are, the richer the bidding strategy should be, with full separation in the limit as they become completely myopic. An additional benefit of a round-bidding strategy is that contemporaneous communication and coordination is not necessary. Firms simply privately observe their costs and bid accordingly on the grid, eliminating logistical concerns about how colluding truckers could manage to coordinate in one hour without knowing who has been invited to the auction.

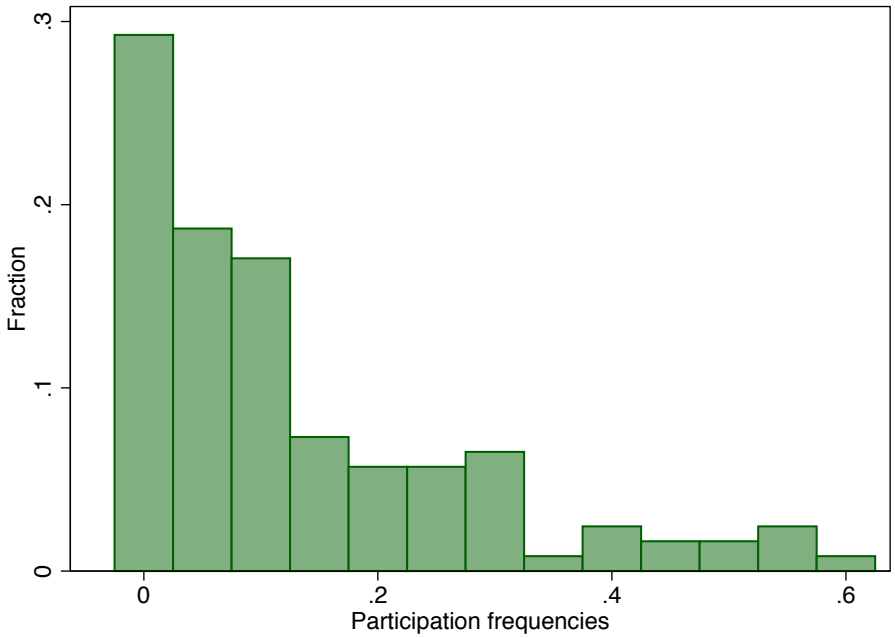
This provides a rationale for why round bidding not only arises empirically, but how it plays a role in solving the complex monitoring and enforcement problems that might otherwise defeat the Association's goals of maximizing its members' profits through the auctions. Importantly, the same tensions exist in the traditional (decentralized) market. Although consumers are free to search for low cost providers by contacting individual truckers or visiting multiple garages, a large fraction of transactions take place through the garage system. At these parking lots, truckers are in relatively close contact and can impose social or economic sanctions on one another for deviating from round bidding, such as refusing to provide assistance for a broken down truck or passing jobs to other truckers as a punishment. This explains why we observe a similar level of price coarseness in the auction platform and in the traditional market.

5.1 Additional evidence of tacit collusion

We interpret the fact that non-competitive behavior is systematic and persistent even for many of the most active bidders as evidence that a majority of bidders intentionally tried to avoid competing in the auctions. In this sub-section we provide additional evidence in favor of this collusion interpretation by analyzing the correlation between our two markers of non-competitive bidding (ties and early bidding), and two additional outcome variables (participation and bid amounts) that are related with efforts to suppress competition. First, we correlate our collusivity index (based off of the propensity to tie) with participation. Second, we look at early bidding and its correlation with the initial and winning bid amounts, especially in revisable-bid auctions.

Participation. In addition to the coarseness and timing of bids, firms can suppress competition by forgoing the opportunity to bid. Figure 8 presents the distribution of participation frequencies across bidders. Indeed we observe that roughly 30% of bidders never bid, and another 30% participate in less than 20% of auctions to which they were invited. Recall that the Association controls operations at the main garages, which potentially allows it to limit competition and keep prices high. One way this can facilitate tacit collusion is by assigning territories to truckers based

Figure 8: Distribution of participation frequencies across bidders



on garage locations. Under this hypothesis, we expect that collusive truckers will be more likely to bid on jobs closer to their garage compared to jobs further away.

Table 8 analyzes the relationship between bidders' collusivity indices (their propensity to tie with the shrinkage correction) and the decision to place a bid in auctions for jobs close to and far from their garage. Appendix Table A-7 presents the results without the shrinkage correction. We test the hypothesis that the correlation between a bidder's collusivity and their participation decision depends on whether the client is near to or far from the bidder's main garage. The dependent variable is an indicator equal to one if the invited firm submitted a bid and the model is estimated by OLS. The columns vary the definition of 'nearness' from 1 to 5 km. We control for bidder fixed effects, so the correlation is identified based on the randomly determined invitations of bidders (as opposed to differences across bidder locations).

We find that bidder type is correlated with the probability that a firm bids for jobs located near their garage. In particular, more collusive bidders are significantly more likely to participate when the job is closer to their garage. This is consistent with the idea that collusive firms are more likely to get business through the garage system, while competitive firms are more likely to work outside of the traditional system; for example they may find clients through referrals, cell phones and street hailing. It is important to note that this result is sensitive to how we measure the collusivity index. The regression coefficients obtained without the shrinkage correction adjusting for measurement error are positive, but not statistically different from zero except for the 2 km distance band (see

Table 8: Relationship between bidders' collusivity index and their participation decision

VARIABLES	(1) 1 km	(2) 2 km	(3) 3 km	(4) 4 km	(5) 5 km
Collusivity index x Dist. $\leq$ Cutoff	0.16^b	0.14^a	0.090^c	0.054	0.012
	(0.074)	(0.052)	(0.048)	(0.049)	(0.048)
Collusivity index x Revisable	0.019	0.020	0.020	0.020	0.020
	(0.017)	(0.017)	(0.017)	(0.017)	(0.017)
Observations	26,279	26,279	26,279	26,279	26,279
R-squared	0.136	0.137	0.137	0.136	0.136
Unit of observation	Invitations	Invitations	Invitations	Invitations	Invitations
Mean variable	0.31	0.31	0.31	0.31	0.31
Nb cluster	40	40	40	40	40

Note: The dependent variable is an indicator for whether that invited firm submitted a bid. The sample is limited to invitations to the 40 most active bidders for whom we have measured a collusivity index. The collusivity index for the bidder is created from their probability of tying in sealed-bid auctions, and takes into account the shrinkage correction. "Dist. $\leq$ Cutoff" is an indicator for the distance between the bidder's garage and the client's home being less than the cutoff in the column heading. Additional controls include: revisable, number of invited bidders (log), distance from client to nearest treatment center, distance from garage to client (1 km bins), client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, arrondissement and day-of-week and bidder fixed effects. Standard errors in parenthesis are clustered at the bidder level. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$.

Appendix Table A-7).

Initial and winning bid amounts. Under the tacit collusion interpretation, firms avoid competing by coordinating on focal prices. In revisable-bid auctions, collusive types can facilitate this coordination by submitting an early bid, which can serve as a reference price for other bidders. An alternative interpretation of early bids is that they are submitted by bidders trying to signal strength by submitting an aggressive first offer, hoping to discourage other bidders from competing. Under some distribution of beliefs, this could in principle rationalize early bidding as a competitive strategy.

Table 9 provides a test for these two signaling stories. In the first column, we measure the difference in the amount of the first bid between sealed-bid and revisable-bid auctions. As before, we control for consumer and auction characteristics. The results show that the first bid received in a revisable-bid auction is 824 CFA higher (roughly 3%) than the first bid in a sealed-bid auction. This is inconsistent with early bidding in revisable-bid auctions being a way to signal strength which would have implied a lower rather than higher bid.

Column (2) looks at the effect of early bidding on the final winning bid. We define a bid as early if it arrives within the first 15 minutes. When this happens in the revisable-bid auction format, all bidders are informed about the value of the lowest early bid, but in sealed-bid auctions no information is provided. Since the presence of an early bid is correlated with the attractiveness

Table 9: Relationship between the presence of early bidding and bid amounts

VARIABLES	(1) First bid amount	(2) Winning bid amount
1(Early bid received)		-0.499[a]
		(0.175)
1(Revisable)	0.824[a]	-0.133
	(0.164)	(0.159)
1(Revisable)x1(Early bid received)		0.750[a]
		(0.245)
Observations	3,627	3,627
R-squared	0.150	0.288
Unit of observation	Auctions	Auctions
Mean dependent variable	27.75	25.73

Note: The indicator variable 1(Early bid received) is equal to one if the first bid received was placed within the first 15 minutes of the auction. The dependent variable in (1) is the bid amount of the first bid received in each auction (in 1,000 CFA), and the dependent variable in (2) is the winning bid amount. Additional controls include: number of invited bidders (log), distance from client to nearest treatment center, average distance from invited bidders' garages to client, client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Robust standard errors in parentheses. *c*: $p < 0.10$, *b*: $p < 0.05$, *a*: $p < 0.01$.

of the job (i.e., attractive jobs receive more bids, and are more likely to receive early bids), we construct a difference-in-differences estimator. In particular, we estimate the difference in the value of the winning bid associated with receiving an early bid in the revisable-bid vs sealed-bid formats. The coefficient associated with the early bid dummy variable is negative and statistically significant (499 CFA), suggesting that early bids and low winning bids are correlated in sealed-bid auctions. Presumably this is because firms bid more aggressively (both lower and more quickly) for jobs they find more attractive in the sealed-bid format. However, the interaction term with the revisable-bid format dummy is positive and large in magnitude (750 CFA), implying that, relative to sealed-bid auctions, consumers are offered significantly higher prices when we observe an early price signal. This result is consistent with the interpretation that early bidding is used by collusive firms in revisable-bid auctions to coordinate on higher bids.

In sum, this section presented supplementary evidence that the patterns we observed in Section 4 are due to collusion, rather than inattention or frictions. Firms with a higher collusivity index are more likely to respect territories, bidding more often close to their garage and less often further afield. Early bids in sealed-bid auctions are associated with lower winning bids as we would have expected in a competitive environment, while early bids in revisable-bid auctions are associated with higher winning bids suggesting they are a coordination mechanism used to collude.

6 Conclusion

We document the importance of imperfect competition and collusion in the market for mechanized desludging services in Dakar. Using non-experimental data on prices and transactions in the traditional desludging market, we first establish that the central districts of Dakar controlled by the Association exhibit 40% higher mechanized desludging prices and 50% lower mechanized desludging take-up than areas in Rufisque supplied by unaffiliated companies. This suggests that collusion may have a large impact on prices which in turn has a large impact on take-up which in turn has deleterious effects on sanitation and health.

We then use experimental data to test for the presence and impact of non-competitive behavior. We created an anonymous auction platform which randomly assigned firms to jobs and auction formats (sealed-bid and revisable-bid). We analyze the bidding and participation strategies of firms to detect deviations from competitive behavior. We document the presence of two main strategies inconsistent with individual profit maximization and competition: (i) the prevalence of ties in the sealed-bid auction format, and (ii) the persistence of early bidding in the revisable-bid auction format. By observing firms randomly invited to bid in the two formats, we establish that a large group of active participants systematically avoid competing by using both of these strategies. In contrast, a smaller group of competitive bidders demonstrate a willingness to undercut their rivals by avoiding ties, and submitting late bids in the revisable-bid auctions.

We conclude that while there exists a group of suppliers willing to submit competitive bids, this group is not big enough to overcome the majority of bidders behaving collusively. This is consistent with what we observe in the traditional market. As Figure 4a illustrates, most consumers in the areas of Dakar controlled by the Association pay very high prices, but a significant fraction of consumers in this region pay prices that are as competitive as in Rufisque. Most of this dispersion in prices is due to unobserved differences across consumers, and is present even within narrowly defined neighborhoods. In light of the experimental results from the auction market, we suspect that some consumers are able to negotiate better prices by getting quotes from non-collusive truckers. As in the auctions, this group of non-collusive truckers is likely too small to serve the entire market. The fact that a majority of firms do not submit competitive quotes, even when it is secret and thus difficult to detect suggests that the fear of losing access to the services and contracts provided by the Association is large enough to discipline most firms active in the market.

Inviting a one standard deviation higher share of the competitive and very competitive firms to participate in an auction would lead to 1.5% lower prices. Although this prediction is out-of-sample, inviting only the very competitive firms is predicted lead to 7.5% lower prices. With high price elasticities of demand for sanitary desludging and the strong effect of manual desludging on diarrhea, this implies that collusion in the desludging market greatly contributes to poor child health outcomes in urban Dakar. This illustrates the importance of market power as a first-order source of market inefficiency. As a result, the welfare effects of eliminating collusion are potentially

very large, since the low take-up of mechanized desludgings has an impact on the health of the entire neighborhood.

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A Appendix tables

Table A-1: Mean differences in characteristics across neighborhoods

	Rest of the market	Rufisque	Difference: Rest - Rufisque	
			Mean	p-value
Nearest center (km)	4.56	2.8	1.77	0
Nearest garage (km)	1.23	.99	.23	0
Num. trucks (3km)	49.64	28.85	20.79	0
Household size	11.19	11.64	-.45	.139
Number of rooms	7.13	7.22	-.09	.592
House ownership	.77	.87	-.09	0
Two story house	.32	.23	.08	0
Wealth index	.04	.17	-.13	.036
Number of other households	1.04	.76	.27	.003
Wide road	.89	.95	-.06	0
Number of earners	3.43	3.21	.22	.055

Table A-2: Regression of mechanized desludging prices on consumer characteristics and distance to the Rufisque boundary

VARIABLES	(1) Rest of the market	(2) Rufisque
1(.5 < Boundary dist. <1)	4.66[a]	2.01[b]
	(1.18)	(0.86)
1(1 < Boundary dist. <1.5)	6.59[a]	0.48
	(0.95)	(0.75)
1(1.5 < Boundary dist. <2)	7.14[a]	3.03[a]
	(0.81)	(0.73)
1(2 < Boundary dist. <2.5)	6.54[a]	2.79[a]
	(0.77)	(0.83)
1(2.5 < Boundary dist. <3)	8.17[a]	1.97[b]
	(0.86)	(0.96)
1(3 < Boundary dist. <3.5)	7.14[a]	2.64[b]
	(0.85)	(1.11)
1(3.5 < Boundary dist. <4)	7.16[a]	
	(1.02)	
1(4 < Boundary dist. <4.5)	7.02[a]	
	(0.96)	
1(4.5 < Boundary dist. <5)	7.54[a]	
	(1.03)	
1(5 < Boundary dist.)	8.23[a]	
	(0.70)	
Nearest center (km)	0.64[a]	-0.40[b]
	(0.088)	(0.20)
Nearest garage (km)	0.16	0.11
	(0.19)	(0.49)
Num. trucks (3km)	0.012[b]	-0.071[b]
	(0.0047)	(0.029)
Household size	-0.038	0.030
	(0.024)	(0.039)
Number of rooms	0.18[a]	0.0094
	(0.042)	(0.076)
House ownership	-0.16	-0.060
	(0.28)	(0.55)
Two story house	1.07[a]	1.34[a]
	(0.26)	(0.47)
Wealth index	0.79[a]	0.14
	(0.096)	(0.17)
Number of other households	-0.021	0.17
	(0.076)	(0.15)
Wide road	1.22[b]	-1.76[c]
	(0.56)	(1.04)
Number of earners	-0.17[a]	-0.15
	(0.062)	(0.11)
Constant	10.1[a]	17.3[a]
	(0.92)	(1.41)
Observations	4,409	394
R-squared	0.122	0.119
H0: Boundary coefficient = 0 (p-value)	0	0.00074
H0: Rufisque coefs. = Rest of market coefs. (p-value)		0

Standard errors in parentheses
[a] p<0.01, [b] p<0.05, [c] p<0.1

Table A-3: Summary statistics on auction outcomes and control variables for sealed-bid and revisable-bid auctions

Variables	Sealed-bid (1) Mean	Sealed-bid (1) SD	Revisable (2) Mean	Revisable (2) SD	Mean difference (1-2)	T-test (1-2)
Bid amount (x1000 CFA)	27.42	5.17	26.97	4.75	0.45	4.24
Dummy: Round bid	0.67	0.47	0.54	0.50	0.13	12.74
Dummy: Bid time > last message	0.15	0.36	0.26	0.44	-0.10	-12.07
Bid time (minutes)	24.34	20.11	29.06	20.86	-4.73	-10.89
Dummy: 0 km $\leq$ Distance < 1 km	0.02	0.12	0.02	0.14	0.00	-1.45
Dummy: 1 km $\leq$ Distance < 2 km	0.04	0.20	0.05	0.21	0.00	-0.74
Dummy: 2 km $\leq$ Distance < 3 km	0.06	0.24	0.06	0.24	0.00	0.08
Dummy: 3 km $\leq$ Distance < 4 km	0.06	0.23	0.06	0.24	-0.01	-1.36
Dummy: 4 km $\leq$ Distance < 5 km	0.05	0.21	0.06	0.23	-0.01	-1.48
Dummy: 5 km $\leq$ Distance < 6 km	0.05	0.23	0.05	0.21	0.01	1.40
Dummy: 6 km $\leq$ Distance < 7 km	0.04	0.20	0.05	0.22	-0.01	-1.60
Dummy: 7 km $\leq$ Distance < 8 km	0.05	0.23	0.05	0.21	0.01	1.40
Dummy: 8 km $\leq$ Distance < 9 km	0.04	0.19	0.03	0.18	0.00	1.20
Dummy: 9 km $\leq$ Distance < 10 km	0.03	0.18	0.03	0.17	0.00	0.39
Number of invited bidders (log)	2.59	0.19	2.59	0.20	0.00	-0.68
Auction hour	11.01	2.14	11.13	2.17	-0.11	-2.47
Distance to nearest treatment center (km)	6.00	2.66	5.99	2.64	0.00	0.04
Average distance to clients (km)	11.37	2.64	11.42	2.86	-0.05	-0.82
Client Latitude coordinate	14.76	0.02	14.76	0.02	0.00	-2.84
Client Longitude coordinate	-17.37	0.05	-17.37	0.05	0.00	-2.62
Arrondissement population (log)	11.19	0.77	11.23	0.76	-0.03	-1.91
Auction count (linear, /1000)	2.17	1.36	2.11	1.34	0.07	2.32
Auction count (quadratic, /1000^2)	6.58	6.93	6.23	6.64	0.35	2.42
Auction count (cubic, /1000^3)	23.40	33.10	21.70	31.30	1.70	2.49
Dummy: New platform	0.30	0.46	0.29	0.45	0.01	1.23
Dummy: Morning	0.65	0.48	0.63	0.48	0.02	2.09
Dummy: Lunch time	0.20	0.40	0.21	0.41	-0.01	-1.53
Dummy: Arrondissement 2	0.01	0.11	0.01	0.10	0.00	0.99
Dummy: Arrondissement 3	0.02	0.14	0.02	0.15	0.00	-0.89
Dummy: Arrondissement 4	0.16	0.37	0.17	0.38	-0.01	-1.65
Dummy: Arrondissement 5	0.21	0.41	0.22	0.41	-0.01	-1.13
Dummy: Arrondissement 6	0.05	0.22	0.04	0.19	0.01	2.68
Dummy: Arrondissement 7	0.11	0.32	0.12	0.33	-0.01	-1.10
Dummy: Arrondissement 8	0.02	0.14	0.02	0.15	0.00	-1.01
Dummy: Arrondissement 9	0.36	0.48	0.34	0.47	0.02	1.94
Dummy: Tuesday	0.20	0.40	0.19	0.39	0.01	0.87
Dummy: Wednesday	0.20	0.40	0.21	0.41	-0.01	-0.85
Dummy: Thursday	0.17	0.37	0.17	0.38	0.00	-0.43
Dummy: Friday	0.17	0.38	0.19	0.39	-0.02	-2.19
Dummy: Saturday	0.03	0.17	0.03	0.17	0.00	0.09
Unit of observation	Bids		Bids			
Number of observations	4487		4432			

Table A-4: Differences in auction outcomes across the two formats

VARIABLES	(1) Winning bid	(2) 1(Ties)	(3) 1(Round)	(4) 1(Last message)	(5) First bid (min.)
1(Revisable)	25.83^a	0.144^a	0.478^a	0.421^a	18.72^a
	(0.0963)	(0.0100)	(0.0118)	(0.0116)	(0.463)
1(Sealed-bid)	25.63^a	0.236^a	0.573^a	0.138^a	15.63^a
	(0.0981)	(0.0119)	(0.0116)	(0.00808)	(0.430)
Observations	3,628	2,503	3,628	3,628	3,628
Unit of observation	Auctions	Auctions	Auctions	Auctions	Auctions
Mean dep. variable	25.73	0.191	0.526	0.279	17.17

Note: The dependent variables are (1) the value of the winning bid, (2) an indicator for the winning bid being tied, (3) an indicator for the winning bid being a multiple of 5,000, (4) an indicator for the winning bid coming in the last ten minutes, and (5) the minute the first bid came in. The sample includes auctions with at least one valid bid in all columns, and with at least two valid bids in column (2). These regression do not include a constant or additional controls. Robust standard errors in parentheses. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$.

Table A-5: Relationship between bidders' collusivity index (without shrinkage correction) and bidding strategies

(a) Sealed-bid auctions

VARIABLES	(1) Bid amount	(2) 1(Round bid)	(3) 1(Late bid)	(4) Bid time
Collusivity index	0.89^a	0.13^a	0.0026	1.47
	(0.18)	(0.0097)	(0.031)	(1.96)
Observations	3,992	3,992	3,992	3,992
R-squared	0.255	0.170	0.054	0.105
Unit of observation	Bids	Bids	Bids	Bids
Mean variable	27.1	0.66	0.16	24.2
Nb cluster	40	40	40	40

(b) Revisable-bid auctions

VARIABLES	(1) 1(Undercut)	(2) 1(Late bid)	(3) Bid time
Collusivity index	-0.052^a	-0.15^a	-5.52^a
	(0.014)	(0.028)	(1.14)
Observations	1,741	3,490	3,490
R-squared	0.083	0.201	0.172
Unit of observation	Bids	Bids	Bids
Mean variable	0.82	0.30	30.9
Nb cluster	40	40	40

Note: The dependent variables are (1a) the value of the bid, (2a) an indicator for the bid being a multiple of 5,000, (3a) an indicator for the bid coming in the last ten minutes, (4a) the minute of the bid, (1b) an indicator for submitting a bid lower than the price information message, (2b) an indicator for the bid coming in the last ten minutes, and (3b) the minute of the bid. Column (1) of panel (b) is limited to bids placed after a price information message in auctions with more than one bid. The collusivity index for the bidder is created from their probability of tying in sealed-bid auctions, and does not take into account the shrinkage correction. Additional controls include: number of invited bidders (log), distance from client to nearest treatment center, distance from garage to client (1 km bins), client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Standard errors clustered at the bidder level in parentheses. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$

Table A-6: Effect of competition and collusion on the acceptance probability

VARIABLES	(1) Accept	(2) Accept	(3) Accept	(4) Accept
Avg. collusivity index		-0.023		-0.030
		(0.021)		(0.048)
V. Competitive (%)	0.21^b		0.074	
	(0.082)		(0.081)	
Competitive (%)	0.072		0.084	
	(0.054)		(0.055)	
Collusive (%)	0.087		0.046	
	(0.060)		(0.056)	
V. Collusive (%)	0.094		0.059	
	(0.075)		(0.076)	
Min. distance (km)	0.0050	0.0052	0.0049	0.0050
	(0.0047)	(0.0047)	(0.0047)	(0.0047)
Min. distance (active)	-0.0069^b	-0.0072^b	-0.0066^b	-0.0070^b
	(0.0030)	(0.0030)	(0.0031)	(0.0030)
Nb. Bidders	-0.0049	-0.0041	-0.0042	-0.0040
	(0.0042)	(0.0042)	(0.0042)	(0.0042)
Nb. Active bidders (< 10 km)	0.0063	0.0059	0.0052	0.0052
	(0.0060)	(0.0059)	(0.0059)	(0.0059)
Nb. Active bidders	0.0011	0.00074	0.0011	0.0010
	(0.0051)	(0.0051)	(0.0051)	(0.0051)
Observations	3,626	3,626	3,626	3,626
R-squared	0.066	0.064	0.065	0.064
Unit of observation	Auctions	Auctions	Auctions	Auctions
Bayesian shrinkage correction	no	no	yes	yes
Mean dependent variable	0.30	0.30	0.30	0.30

Note: The dependent variable in each specification is an indicator variable equal to one if the client accepts the offer. The regression is estimated by OLS. The collusion/competition categories are only available for active bidders and are defined as (i) very competitive ($\theta_i \leq \bar{\theta}_{0.1}$), (ii) competitive ($\bar{\theta}_{0.1} < \theta_i \leq \bar{\theta}_{0.3}$), (iii) median bidder ($\bar{\theta}_{0.3} < \theta_i \leq \bar{\theta}_{0.7}$), (iv) collusive ($\bar{\theta}_{0.7} < \theta_i \leq \bar{\theta}_{0.9}$), and (v) very collusive ($\bar{\theta}_{0.9} < \theta_i$). Where $\bar{\theta}_q$ denotes the qth quantile of the empirical distribution of θ_i. The median group is omitted from the regression. Additional control variables include: number of invited bidders (log), distance from client to nearest treatment center, average distance from invited bidders' garages to client, client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, and arrondissement and day-of-week fixed effects. Robust standard errors in parentheses. *c*: $p < 0.10$, *b*: $p < 0.05$, *a*: $p < 0.01$.

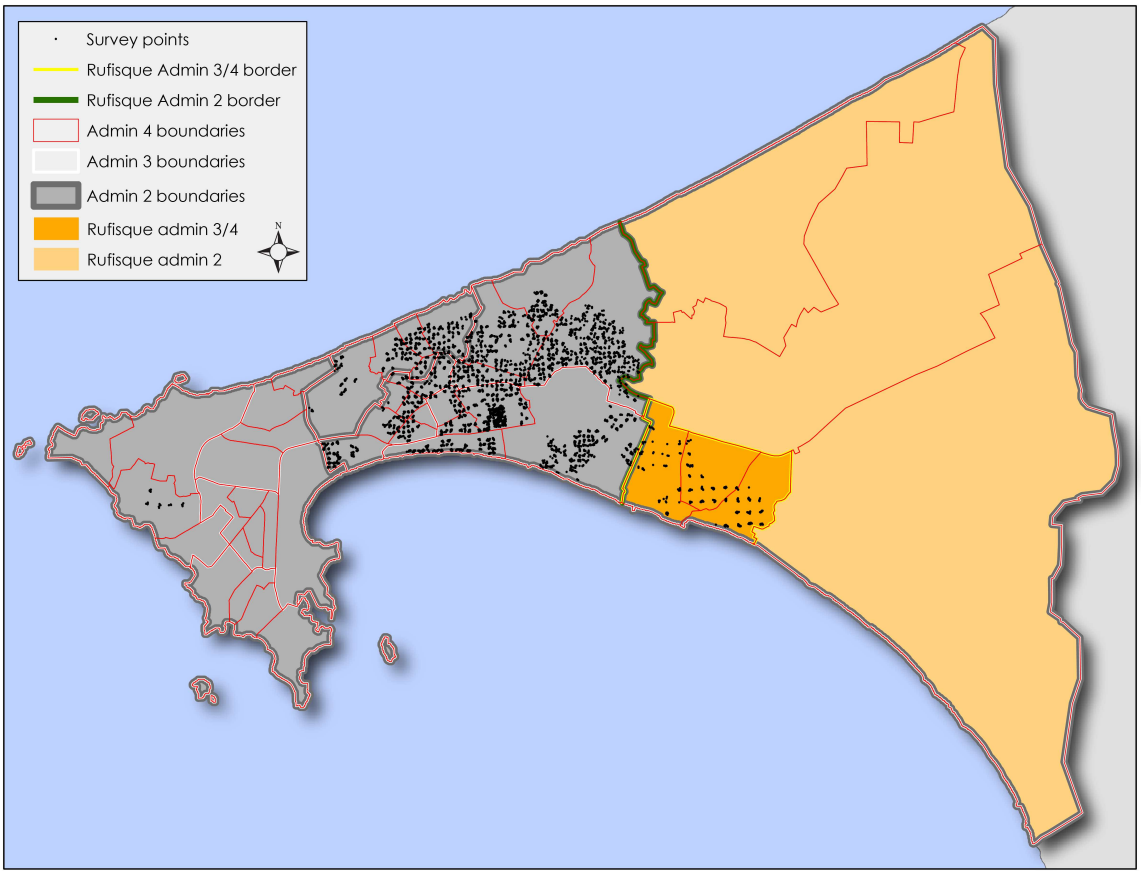
Table A-7: Relationship between bidders' collusivity index (without shrinkage correction) and their participation decision

VARIABLES	(1) 1 km	(2) 2 km	(3) 3 km	(4) 4 km	(5) 5 km
Collusivity index x Dist. $\leq$ Cutoff	0.028	0.080^b	0.038	0.023	0.0098
	(0.054)	(0.035)	(0.029)	(0.027)	(0.024)
Collusivity index x Revisable	0.011^c	0.011^c	0.011^c	0.012^c	0.011^c
	(0.0059)	(0.0058)	(0.0059)	(0.0058)	(0.0059)
Observations	26,279	26,279	26,279	26,279	26,279
R-squared	0.136	0.137	0.136	0.136	0.136
Unit of observation	Invitations	Invitations	Invitations	Invitations	Invitations
Mean variable	0.31	0.31	0.31	0.31	0.31
Nb cluster	40	40	40	40	40

Note: The dependent variable is an indicator for whether that invited firm submitted a bid. The collusivity index for the bidder is created from their probability of tying in sealed-bid auctions, and does not take into account the shrinkage correction. "Dist. $\leq$ Cutoff" is an indicator for the distance between the bidder's garage and the client's home being less than the cutoff in the column heading. Additional controls include: number of invited bidders (log), distance from client to nearest treatment center, distance from garage to client (1 km bins), client latitude and longitude, auction number trend (linear, quadratic and cubic), new platform indicator, auction hour, indicators for morning and lunch time, arrondissement and day-of-week and bidder fixed effects. Standard errors clustered at the bidder level are in parentheses. c: $p < 0.10$, b: $p < 0.05$, a: $p < 0.01$.

B Appendix figures

Figure B-1: Distribution of survey households across neighborhoods



C Round bidding as optimal collusion

Literature. The model we study is closest to Athey et al. (2004), and to a lesser extent Horner & Jamison (2007), who examine a repeated Bertrand pricing game. They come to similar conclusions about 'rigid pricing.' In the presence of incomplete information about costs, a Cartel can benefit by eschewing full separation in favor of a bidding scheme in which firms collude on the monopoly price. They focus on the case in which firms are sufficiently patient to collude on the monopoly price, and consider a second case in which two prices are used. In contrast, we explicitly consider auctions rather than markets and allow for many rigid bids, rather than just one or two. The previous papers study the optimality of the agreement as posed as a problem of imperfect public monitoring, while we study the existence of collusive equilibria in the auction supergame that exhibit the empirical

patterns observed in Senegal. In particular, we show that most profitable round-bidding strategies exist even for levels of patience that do not support rigid bidding on the optimal monopoly price (here, the reserve price).

In many ways, this is really a commitment problem as considered in Skreta (2006) and Doval & Skreta (2022). The Cartel would like to commit its members to a certain pattern of behavior, but the members cannot themselves commit to follow it. These problems are typically studied from the perspective of a seller facing a group of buyers (to what extent can an auctioneer credibly refuse 'lowball' bids below a reserve price?). Our setting studies the perspective of a buyer facing a group of sellers (if the Cartel members can privately deviate ex post from a prescribed pattern of bidding, how can the Cartel maintain discipline ex ante?). In this sense, we find the commitment framework to be more useful than the repeated games framework.

A classic literature in auction theory following McAfee & McMillan (1992) considers how to design collusive bidding rings. This literature typically maximizes the Cartel's expected revenue, not the expected payoffs of the individual members, which avoids the issues of commitment and monotonicity that appear later. The Cartel member with the lowest cost is the one that maximizes Cartel revenue, so the issue in that literature is how to compensate the other members of the Cartel for 'taking a dive.' However, McAfee & McMillan (1992) also note that for weak cartels when side payments aren't feasible, there should arise price rigidity such that firms all collude on a single bid. Laffont & Martimort (1997) consider a version of the problem that explicitly assumes commitment to the mechanism, but generalizes other aspects of the analysis in McAfee & McMillan (1992). Che & Kim (2007) study the problem from the platform's perspective, and conclude that a simple take-it-or-leave-it-offer from the platform to the Cartel is optimal. The main distinction in our case is that we study equilibrium behavior of the Cartel to verify the plausibility of our empirical observations in Senegal, as opposed to studying how the platform can best combat the presence of collusive participants.

To fix ideas and maintain verisimilitude with the auctions in Senegal, consider a paid-as-bid format in which the lowest bidder wins and is paid their bid, as long as that bid is below a reserve price R. Then in a non-collusive Bayesian Nash equilibrium, firms would maximize their private profits,

$$\pi_i(c_i) = \max_{b_i}(b_i - c_i) \times \text{pr}[\text{Submitted the lowest bid less than } R \mid b_i].$$

The reason to focus attention on the paid-as-bid mechanism in particular is that the platform must be budget-balanced, so that transfers from the household to the desludger must be equal for each job.

Incentive compatibility. In a Direct Mechanism, bidders report a type $\hat{c}_i$ rather than directly place a bid, and then the Cartel itself maps reports to probabilities of winning $p(\hat{c}) = \{p_i(\hat{c}_1, ..., \hat{c}_N)\}_{i=1}^N$ and payments received $b(\hat{c}) = \{b_i(\hat{c}_1, ..., \hat{c}_N)\}_{i=1}^N$ that are consistent with the

underlying paid-as-bid auction. So while the platform picks the winner at random from the set of bidders submitting the lowest bid less than the reserve price, the Cartel can manipulate who submits which bids and their values, reflected in $\langle p, b\rangle$.

From firm i's perspective, the *interim probability* that it wins given a report of $\hat{c}_i$ is

$$p_i(\hat{c}_i) = \mathbb{E}_{\hat{c}_{-i}}\left[p_i(\hat{c}_1, ..., \hat{c}_{i-1}, \hat{c}_i, \hat{c}_{i+1}, ..., \hat{c}_N)\right]]$$

where $p_i(\hat{c}_1, ..., \hat{c}_N)$ is the probability that the Cartel selects firm i to win given the reports $\hat{c}$. Before asking what kinds of pairs $\langle b, p\rangle$ are optimal for the Cartel, the more fundamental question is, given that the bidders can lie about their types, what kinds of pairs $\langle p, b\rangle$ are actually possible, or *implementable*? For what kinds of $\langle p, b\rangle$ do bidders submit $\hat{c}_i = c_i$?

The classic characterization of Bayesian incentive compatibility is:

Proposition 1 (Myerson (1981))**.** *A Direct Mechanism $\langle p, b\rangle$ is Bayesian incentive compatible iff it satisfies the monotonicity and envelope conditions. Firm i's interim payoff is*

$$\pi_i(c_i) = \max_{\hat{c}_i} p_i(\hat{c}_i)(b_i(\hat{c}_i) - c_i) = \int_{c_i}^{R} p_i(z)dz,$$

and equilibrium bid function is

$$b(c_i) = c_i + \frac{\int_{c_i}^{R} p_i(z)dz}{p_i(c_i)}. \tag{5}$$

A key assumption is that the Cartel can commit to the mechanism. For example, in an auction to sell a good, the auctioneer can commit to refuse to consider bids below the reserve price. This will become relevant in our environment: the extent to which the Cartel can enforce discipline is limited by its ability to detect and punish deviations.

Static platform and Cartel optimization. The Cartel is a collective of firms that maximize their expected interim profits,

$$\mathbb{E}_{(c_1,...,c_N)}\left[\sum_{i=1}^{N} \pi_i(c_i, c_{-i})\right].$$

Before turning to the Cartel's solution, it helps to revisit the platform's problem and the standard non-collusive solution. In standard auction design without a colluding Cartel, Proposition 1 is sufficient to derive the platform's expected payoff for any Bayesian incentive compatible mechanism $\langle p, b\rangle$. In particular, rearranging Equation (5) allows the expected payment for the c_i type to be expressed as

$$p_i(c_i)b_i(c_i) = p_i(c_i)c_i + \int_{c_i}^{R} p_i(z)dz.$$

Taking the expectation over c_i yields

$$\mathbb{E}_c[p_i(c)b_i(c)] = \int_{\underline{c}}^{R} \left(p_i(c_i)c_i + \int_{c_i}^{R} p_i(z)dz \right) dG(c_i) + \int_{R}^{\bar{c}} (0)dG(c_i)$$

and integration by parts with respect to c_i yields the expected payment to firm i,

$$\mathbb{E}_c[p_i(c_i)b_i(c_i)] = \mathbb{E}_c \left[p_i(c_1, ..., c_N) \left(c_i + \frac{G(c_i)}{g(c_i)} \right) \right].$$

The quantities

$$\phi(c_i) = c_i + \frac{G(c_i)}{g(c_i)} \quad \text{and} \quad \lambda(c_i) = \frac{G(c_i)}{g(c_i)}$$

are the *virtual cost* and *informational rent*, respectively. The informational rent captures the portion of the expected cost of the job that accrues to the firm as compensation for revealing its private information, and the virtual cost is the true cost plus the informational rent.

Let v be the value to the household of getting the job done (or the platform's estimate of that value). Then the platform deducts the expected procurement cost for each firm from the expected benefit and takes the weighted sum over firms to get expected Consumer Surplus:

$$\mathbb{E}_{(c_1,...,c_N)} \left[\sum_{i=1}^{N} p_i(c_1, ..., c_N) \times (v - \phi(c_i)) \right].$$

How does the platform maximize this quantity? Setting aside the monotonicity condition for the moment, the optimal choice is to pick the firm i for whom $v - \phi(c_i)$ is largest and non-negative, and refuse to award the job if the virtual cost exceeds the benefit for every firm. When is the monotonicity condition slack? As long as $\phi(c_i)$ is non-decreasing, so that the firm with the lowest virtual cost is also the firm with the lowest realized cost. Then a first-price or second-price auction with a reserve price satisfying $\phi(R) = v$ will be an optimal mechanism, as well as more exotic games like the all-pay auction or third-price auction. This is the general logic of the celebrated Revenue Equivalence result. Equation (5) determines payments as a function of the probabilities of winning, so that all Bayesian incentive compatible mechanisms that award the good the same way generate the same expected Consumer Surplus.

In order that the monotonicity condition be satisfied however, so that the optimal allocation rule $p(c)$ selects the firm with the lowest reported cost to do the job, it is necessary that

$$\phi(c_i) = c_i + \frac{G(c_i)}{g(c_i)}$$

be non-decreasing, and sufficient that

$$\frac{G(c_i)}{g(c_i)}$$

be a non-decreasing function; essentially, that $G(c_i)$ is log-concave.[28] Otherwise, the platform would want to violate the monotonicity condition's restriction that higher costs win with lower probability, and this would not be implementable. The lower-cost firm would simply pool with the higher-cost firm who is winning with higher probability, thereby increasing its payoff.

All of this is familiar auction design, but from the platform's perspective, the goal is approximately to minimize the expected cost of the job. The Cartel, however, has almost the opposite objective: to maximize expected interim profit. Indeed, if firm i's expected interim profit is

$$\pi_i(c_i) = \int_{c_i}^{R} p_i(z)dz + \pi_i(R)$$

where $\pi_i(R) = 0$, then a similar set of calculations as above yields

$$\mathbb{E}_{c_i}[\pi_i(c_i)] = \mathbb{E}_{(c_1,...,c_N)}\left[p_i(c_1,...,c_N)\frac{G(c_i)}{g(c_i)}\right],$$

and summing over the firms yields the Cartel's interim profits:

$$\mathbb{E}_c\left[\sum_{i=1}^{N}\pi_i(c_i)\right] = \mathbb{E}_{(c_1,...,c_N)}\left[\sum_{i=1}^{N}p_i(c_1,...,c_N)\lambda(c_i)\right].$$

The Cartel is maximizing its expected informational rents, and has preferences that are approximately opposed to those of the platform. Unconstrained, it would like to pick the winner from among the firms with the highest realized costs, since $\lambda(c_i)$ is non-decreasing under the standard regularity condition. This is not implementable for a simple reason: every firm would then report that it was the highest type that wins with positive probability to the Cartel in order to maximize its payment and probability of being selected to bid and win in the auction. Consequently, every Cartel-optimal solution will involve pooling when $\lambda(c_i)$ is non-decreasing.

Consider mechanisms in which the Cartel partitions $[\underline{c}, R]$ by a set of interior points $\mathcal{C} = \{c_1, c_2, ..., c_L\}$, with the notational convention that $c_0 = \underline{c}$ and $c_{L+1} = R$. To each sub-interval there corresponds a bid: $[\underline{c}, c_1)$ to b_1, $[c_1, c_2)$ to b_2, ..., and $[c_L, R]$ to R. Denote these bids by $\mathcal{B}$. We call this kind of mechanism a *round-bidding strategy.* Ultimately, focusing on round-bidding strategies is without loss of generality, because as many interior types are added to the same interval, the probability of a tie in that interval tends to zero, and strict separation occurs in the limit.

Proposition 2. *If $\lambda(c_i)$ is non-decreasing on $[\underline{c}, R]$, then the optimal mechanism for the Cartel is a round-bidding strategy. Optimal mechanisms exist.*

Any mechanism with a strictly separating interval can be improved upon: by using the ironing procedure of Mussa & Rosen (1978) and Myerson (1981) and pooling those types into one report,

[28] Since $D^2\log(G) = (g'G - g^2)/G^2$ and $D(G/g) = (g^2 - g'G)/g^2$, $-D^2\log(G)$ and $D(G/g)$ have the same sign. So G/g is non-decreasing if G is log-concave.

the expected informational rents from that interval increase. Full separation on any sub-interval is sub-optimal, since some amount of pooling can reduce competition and raise expected informational rents.

The intuition for this result is that the informational rent $\lambda(c_i)$ is a non-decreasing function, but any incentive compatible mechanism must have a non-increasing probability of winning.

Dynamic enforcement and round bidding. Can the Cartel achieve the static optimum in a repeated game when monitoring and enforcement are required? A key assumption of the static analysis is that the Cartel can *commit to the mechanism* (e.g., Doval & Skreta (2022), Skreta (2006)). For an example of commitment, in an auction to sell a good the auctioneer can threaten to refuse to consider bids below the reserve price but this might not be credible in practice. In this environment, a similar issue becomes relevant: the firms do not report to the Cartel who bids for them, they place bids themselves, and will be tempted to renege on the Cartel's agreement once they know their private information. The extent to which the Cartel can enforce discipline is therefore limited by its ability to detect and punish deviations. Bids are not submitted to the Cartel directly, but to the platform, and the individual Cartel members' bids are largely unobservable to the Cartel as a whole. If all Cartel members are supposed to adopt the round-bidding strategy or withdraw when their costs exceed the reserve price, then low-cost types will be tempted to undercut round bids and win with strictly higher probability and make arbitrarily similar payments. If this kind of deviation isn't somehow monitored and punished, Cartel discipline crumbles.

With fully separating equilibria and public monitoring of just the winning bid, every history is approximately (up to withdrawals by firms with costs above the reserve price) measure zero, so deviations can only be inferred from examining the statistical properties of long histories to detect anomalies. With round-bidding strategies, however, compliance can be directly observed from the history of the winning bid: if it is on-$\mathcal{B}$, the Cartel has followed the agreement, and if any off-$\mathcal{B}$ winning bids are observed, someone has necessarily cheated. Thus, unlike with fully separating equilibria, deviations can be detected and punished instantly with round bidding.

Let the firms' common discount factor be given by δ. Then $\mathcal{B}$ is *enforceable for* δ if there exists some set of strategies in the infinitely repeated game for which there are no profitable deviations from bidding on-$\mathcal{B}$ after any history. The static unprofitability of strict separation on any interval actually facilitates dynamic enforcement.

Theorem 3. *If the discount factor δ is sufficiently close to 1, any incentive compatible round-bidding strategy $\mathcal{B}$ is enforceable. For a fixed discount factor δ, there exists a most profitable incentive compatible round-bidding strategy $\mathcal{B}^*$ that is enforceable for δ. As δ increases, the optimal round-bidding strategy has weakly fewer bids.*

So when the Cartel cannot commit to a mechanism, the optimal grid $\mathcal{B}$ becomes a function of temptation as well as collusive motives. Relatively impatient Cartels can reduce the benefits

of deviating and undercutting high on-$\mathcal{B}$ bids by introducing more bid increments, compensating compliance with a higher probability of winning. This keeps competition soft relative to the fully separating equilibrium, but reduces the temptation for lower cost firms to deviate. As the discount factor goes to zero, only the fully separating equilibrium is incentive compatible and enforceable as the limit of a round-bidding strategy with infinitely many increments, and as the discount factor goes up to one, any incentive compatible round-bidding strategy is enforceable. For intermediate values of δ, the optimal $\mathcal{B}$ will be finite but become denser as firms become more impatient. This rationalizes the existence of relatively rich but finite grids, when the static solution suggests an extremely coarse one that includes only the reserve price R and the non-cooperative Bayesian Nash equilibrium has full separation.

It is surprising that no coordination or communication by the Cartel is required for optimal collusion. The pooling intervals are invariant to the realization of the firms' cost types, so the optimal Cartel does not need to run a pre-auction knockout or otherwise exploit idiosyncratic and transient private information to determine the winner. Likewise, there is no 'turn-taking' or 'phases of the moon' coordination that might be detected using forensic algorithms. Firms bid in what appears to be an uncoordinated and competitive fashion, hiding their collusive behavior in round bidding that can be shrugged off as boundedly rational behavior. In the context of Senegal, this is essential, because it means the Cartel does not actually have to coordinate its behavior from auction to auction on tight deadlines: truckers optimally coordinate by using the round-bidding scheme.

Secret reserve prices. The model used a fixed reserve price R and subsequent analysis assumed that the household's willingness-to-pay W was known to the platform, which presumably set the optimal reserve price in the standard way, as $\phi(R) = w$. In reality, the households' willingness-to-pay vary from job to job and are unknown to the platform in advance. One alternative way to model the household's accept-or-reject decision is as a secret reserve price: if the distribution function for the household's willingness-to-pay W is $F_W(w)$, then the probability a bid b_i is accepted is $R(b_i) = 1 - F_W(b_i)$.

This creates an additional margin that complicates the Cartel's problem. Higher bids are now more likely to be rejected, so that colluding on a high price is not necessarily profit maximizing, since there is a continuous trade-off between the probability of a bid being accepted and job remuneration, as opposed to a single reserve price where the probability the bid is accepted goes from one to zero.

The presence of a secret reserve price thereby impacts the structure of the optimal partition. When the Cartel's problem is locally concave because the secret reserve price and likelihood of rejection dominate, there will be separation in bidding as above, and when the Cartel's problem is locally convex and the non-decreasing information rent dominate, it will employ pooling. So the exact analytical properties of the optimal solution can be determined from an optimal control approach, but the qualitative properties are similar.

Proof of Proposition 2:

Proof. Consider some candidate non-increasing $p^*(c)$ function. Since $p^*(c)$ is a non-increasing and bounded function on a compact interval, it has at most a countable number of jump discontinuities. Take any interval (a, b) on which $p^*(c)$ is continuous and strictly decreasing.

Since the mechanism is fully separating on this interval and incentive compatible, the probability of winning can be decomposed into the probability of having the lowest type, multiplied by the conditional probability of winning at all, conditional on the reported type, $\alpha(c_i)$,

$$(1 - G(c_i))^{N-1}\alpha(c_i).$$

Since p^* is continuous on (a, b), $\alpha(c_i)$ must then be continuous as well.

Consider ironing this interval, so that all types in (a, b) make the same report and receive the same probability of winning. The probability of being randomly selected as the winner given conditional on reporting a type in this interval is

$$\bar{p} = \sum_{j=0}^{N-1} \frac{1}{j+1} \frac{(N-1)!}{j!(N-1-j)!}(1 - G(b))^{N-1-j}(G(b) - G(a))^j.$$

This can be rewritten as

$$\begin{aligned} N(G(b) - G(a))\bar{p} &= \sum_{j=0}^{N-1} \frac{N!}{(j+1)!(N-(j+1))!}(1 - G(b))^{N-(j+1)}(G(b) - G(a))^{j+1} \\ &= \sum_{j=1}^{N} \frac{N!}{j!(N-j)!}(1 - G(b))^{N-j}(G(b) - G(a))^j \end{aligned}$$

Note that the right-hand side is similar to the distribution of an order statistic, except for the truncation at $G(a)$. The standard induction arguments provided in Proposition 4 below, however, reduce it to the quantity

$$H(a, b) = \int_a^b N(1 - G(x))^{N-1} g(x)dx,$$

so that

$$\bar{p} = \frac{\int_a^b N(1 - G(x))^{N-1} g(x)dx}{N(G(b) - G(a))} = \frac{\int_a^b (1 - G(x))^{N-1} g(x)dx}{G(b) - G(a)}. \tag{6}$$

Notice that this quantity can be written

$$\bar{p} = \mathbb{E}[(1 - G(x))^{N-1} | a < x < b].$$

Now, by the mean value theorem, there exists an $\bar{\alpha} = \alpha(\xi), \xi \in (a, b)$, such that

$$\int_a^b (1 - G(x))^{N-1} \alpha(x)\lambda(x)g(x)dx = \int_a^b (1 - G(x))^{N-1} \lambda(x)g(x)dx\bar{\alpha}.$$

Imagine ironing (a, b), and giving those who bid a $c_i \in (a, b)$ the probability $\bar{p}\bar{\alpha}$ of winning, which is feasible with respect to the monotonicity condition because $p^*(a)\alpha(a) \geq \bar{p}\bar{\alpha}$. Now consider the comparison

$$\int_a^b (1 - G(x))^{N-1} \alpha(x)\lambda(x)g(x)dx \gtreqless \int_a^b \bar{p}\bar{\alpha}\lambda(x)g(x)dx,$$

or, by previous calculations, this also equals

$$\int_a^b (1 - G(x))^{N-1} g(x)\lambda(x)dx \gtreqless \int_a^b \bar{p}\lambda(x)g(x)dx,$$

or

$$\int_a^b \frac{(1 - G(x))^{N-1}}{\bar{p}} \frac{g(x)}{G(b) - G(a)} \lambda(x)dx \gtreqless \int_a^b \frac{g(x)}{G(b) - G(a)} \lambda(x)dx.$$

Considering the left-hand side of the inequality, define

$$w(x) = \frac{(1 - G(x))^{N-1}}{\bar{p}} \frac{g(x)}{G(b) - G(a)}.$$

Then $\int_a^b w(x)dx = 1$, since $\bar{p} = \mathbb{E}[(1-G(x))^{N-1}|a < x < b]$ is the conditional mean of $(1-G(x))^{N-1}$ on (a, b). Likewise, $\int_a^b dG(x)/(G(b)-G(a)) = 1$ on the right-hand side. But $w(x)$ places more weight on low values of x compared to $g(x)/(G(b) - G(a))$, and less weight on high values of x. Since $\lambda(x)$ is non-decreasing,[29] that implies the left-hand side must be weakly less than the right-hand side, and ironing the interval is weakly profitable. Ironing this way is feasible since it does not violate any monotonicity constraints, and indeed relaxes them on subsequent intervals of the type space. Therefore, ironing any continuously separating interval raises the value of the objective without violating any of the constraints.

Therefore, we can restrict attention to mechanisms in which there are no strictly decreasing segments, and $p^*(c)$ is constant almost everywhere and non-increasing.

[29] Note that if $\lambda(x)$ were decreasing, as $v-\phi(x)$, we would have the standard, opposite conclusion that full separation is optimal.

Optimal partitioning. Take any grid $\mathcal{C} \subseteq [\underline{c}, R]$. The probability of winning when reporting a type $c_i \in [c_{\ell-1}, c_\ell)$ is

$$p_\ell = p(c_{\ell-1}, c_\ell) = \sum_{j=0}^{N-1} \underbrace{\frac{1}{1+j}}_{\text{Tie-breaking}} \times \underbrace{\frac{(N-1)!}{j!(N-1-j)!} \times (G(c_\ell) - G(c_{\ell-1}))^j (1 - G(c_\ell))^{N-1-j}}_{\text{Probability of } j \text{ other bidders in the } \ell\text{-th bin, } N-1-j \text{ in strictly higher bins}} .$$

This depends only on $c_{\ell-1}$ and c_ℓ, which was previously shown to equal

$$p(c_{\ell-1}, c_\ell) = \frac{H(c_{\ell-1}, c_\ell)}{N(G(c_\ell) - G(c_{\ell-1}))}.$$

For a fixed L, a partition $\mathcal{C}$ that satisfies the monotonicity constraint exists since c_L can be selected to be very close to R, c_{L-1} very close to c_L, and so on. The probabilities that the higher elements of the partition win are arbitrarily small and the entire $\{p_\ell\}_{\ell=1}^L$ sequence is non-increasing. This is a feasible point for any finite L, so the feasible set is non-empty. Call any such $\mathcal{C}$ *admissible*.

Take any admissible $\mathcal{C}$. Then it determines $\mathcal{B}$ because we can write a set of indifference conditions for all of the boundary types that must get the same payoff from b_ℓ and $b_{\ell+1}$,

$$p_\ell(b_\ell - c_\ell) = p_{\ell+1}(b_{\ell+1} - c_\ell), \quad \ell = 1, ..., L$$

with terminal condition

$$p_L(b_L - c_L) = p_R(R - c_L).$$

The terminal condition can be solved for b_L in terms of R and c_L,

$$b_L = c_L + \frac{p_R}{p_L}(R - c_L),$$

and then back-substituted through the rest of the conditions for a fixed $\mathcal{C}$ and $b_{\ell+1}$ to get b_ℓ, using the recursion

$$b_\ell = c_\ell + \frac{p_{\ell+1}}{p_\ell}(b_{\ell+1} - c_\ell).$$

The bids are therefore a continuous function of the type partition, $\mathcal{B}(\mathcal{C})$, and the mapping between them means each is entirely determined by the other.

Let

$$H(c_{\ell-1}, c_\ell) = \sum_{j=1}^{N} \frac{N!}{j!(N-j)!} (G(c_\ell) - G(c_{\ell-1}))^j (1 - G(c_\ell))^{N-j}$$

be the probability that the lowest of N draws is between $c_{\ell-1}$ and c_ℓ. Then — using the same calculations as for (6) to express $p(c_{\ell-1}, c_\ell)$ in terms of $H(c_{\ell-1}, c_\ell)$ by way of Proposition 4 — the

Cartel's expected payoff in a round in which all firms participate as intended is

$$N \sum_{\ell=1}^{L+1} p(c_{\ell-1}, c_\ell) \int_{c_{\ell-1}}^{c_\ell} \lambda(x) g(x) dx = \sum_{\ell=1}^{L+1} H(c_{\ell-1}, c_\ell) \mathbb{E}[\lambda(c) | c_{\ell-1} < c < c_\ell] \tag{7}$$

where the monotonicity constraint is[30]

$$p_\ell = \frac{H(c_{\ell-1}, c_\ell)}{N(G(c_\ell) - G(c_{\ell-1}))} \geq \frac{H(c_\ell, c_{\ell+1})}{N(G(c_{\ell+1}) - G(c_\ell))} = p_{\ell+1}, \quad \ell = 1, ..., L,$$

and the bids $\mathcal{B}(\mathcal{C})$ are determined by the backwards recursion

$$b_\ell = c_\ell + \frac{p_{\ell+1}}{p_\ell}(b_{\ell+1} - c_\ell), \quad \ell = 1, ..., L$$

with boundary condition $b_{L+1} = c_{L+1} = R$.

Since the objective function is continuous in $\mathcal{C}$ and the constraints are all continuous weak inequalities, the constraint set is a closed subset of $[\underline{c}, \overline{c}]^L$ and therefore compact. Therefore, a solution to the problem exists for a fixed L, by the extreme value theorem.

Existence of a finite, optimal L. The previous work established existence of an optimal partition $\mathcal{C}_L^*$, but not that there is a finite L that corresponds to a solution. In principle, the benefit of adding additional sub-intervals to the mechanism might increase indefinitely, so that no solution exists. The next part of the proof shows this is not the case.

Define the *widest implementable partition* for L as the solution to

$$J_L^W = \max_{\mathcal{C}} \min_{\ell} (c_{\ell-1} - c_\ell)^2$$

subject to $p(c_{\ell-1}, c_\ell) \geq p(c_\ell, c_{\ell+1})$ for $\ell = 1, ..., L$. Just like the interim profit maximization problem, this one has a solution because the objective function is continuous and the constraints characterize a closed subset of a compact set. This partition will maximize the minimum distance between its elements in the type space, subject to being implementable. Note that this implies that any other implementable partition $\mathcal{C}$ for any finite L must have some interval on which points are closer together, by construction. The solution to this problem spaces the points of $\mathcal{C}_L^W$ as far apart as possible without violating incentive compatibility. As shown earlier, a solution exists for any fixed

[30] The quantity $\frac{H(c_{\ell-1}, c_\ell)}{N(G(c_\ell) - G(c_{\ell-1}))}$ looks like it might be poorly behaved (discontinuous, singular, etc.) as $c_\ell \to c_{\ell-1}$, but this isn't the case, because

$$\lim_{c_\ell \downarrow c_{\ell-1}} p(c_{\ell-1}, c_\ell) = \lim_{c_\ell \downarrow c_{\ell-1}} \frac{\int_{c_{\ell-1}}^{c_\ell} N(1 - G(x))^{N-1} g(x) dx}{N(G(c_\ell) - G(c_{\ell-1}))} = \lim_{c_\ell \downarrow c_{\ell-1}} \frac{\int_{c_{\ell-1}}^{c_\ell} N(1 - G(x))^{N-1} g(x) dx / (c_\ell - c_{\ell-1})}{N(G(c_\ell) - G(c_{\ell-1}))/(c_\ell - c_{\ell-1})} = (1 - G(c_{\ell-1}))^{N-1},$$

which is correct as the partition shrinks down to a single point.

L, so $\mathcal{C}_L^W$ and J_L^W are well-defined for all $L \in \mathbb{N}$.

Consider what happens as L becomes large: the optimal partitions $\mathcal{C}_L^W$ include more points that are partitioning the same bounded set, so that J_L^W must be a non-increasing sequence bounded below by a limit of zero. The probability of a tie then converges to zero almost everywhere, since the odds of another firm being in the same sub-interval $[c, c+\Delta)$ goes to zero:

$$\lim_{\Delta \downarrow 0} p_\Delta(c) =$$
$$\lim_{\Delta \downarrow 0} \left\{ (1-G(c+\Delta))^N + \sum_{j=1}^{N-1} \frac{1}{1+j} \times \frac{(N-1)!}{j!(N-1-j)!} \times (G(c+\Delta)-G(c))^j (1-G(c+\Delta))^{N-1-j} \right\}$$
$$= (1-G(c))^N.$$

But this implies that $\mathcal{C}_L^W$ gets arbitrarily close to implementing the assignment rule

$$\overline{p}_i(c) = \begin{cases} \dfrac{1}{|\{k : c_k = \min_j c_j|\}|}, & i \in \operatorname{argmin}_j c_j \\ 0, & \text{otherwise} \end{cases}$$

which is the fully-separating solution. That is the decision rule that minimizes the Cartel's objective function by minimizing informational rents, so this limit mechanism cannot be optimal. More formally, for any $\varepsilon > 0$, there exists a $\tilde{L}$ and a sub-interval of $[\underline{c}, R]$ of strictly positive measure where the difference in the supnorm between the assignment rule based on $\mathcal{C}_L^W$ and $\overline{p}_i(c)$ is less than ε, and on that particular sub-interval there is a strictly profitable deviation for the Cartel that involves coarsening the partition to induce more pooling and less separation.

But $\mathcal{C}_L^W$ is the partition that spaces the points apart as widely as possible. Any other sequence — particularly the one that corresponds to the optimal partition with L points for each L — is going to exhibit the same phenomenon because it is not constructed to space the points as widely as it can. For any $\varepsilon > 0$, there will be some sub-interval of $[\underline{c}, R]$ where the points are clustered together at least as tightly as $\mathcal{C}_L^W$, and are even closer to the fully separating limit. Therefore, the optimal sequence of partitions $\mathcal{C}_L^*$ for L sufficiently large must have sub-intervals that include even more points than $\mathcal{C}_L^W$, and are even closer to full separation. For the same reason, this will result in sub-intervals that are arbitrarily close to fully-separating as L becomes arbitrarily large, which cannot be optimal if the Cartel is free to reduce L and adopt a coarser partition.

Define

$$J(L) = \max_{\mathcal{C}} \sum_{\ell=1}^{L+1} H(c_{\ell-1}, c_\ell) \mathbb{E}[\lambda(c) | c_{\ell-1} < c < c_\ell].$$

Then there must be some finite $\overline{L}$ such that for $L' > \overline{L}$, $J(\overline{L}) > J(L')$. Then since the set $\{1, ..., \overline{L}\}$ is compact, a solution to the problem $\max_{L \in 1,...,\overline{L}} J(L)$ must exist, which corresponds to the optimal

mechanism.

□

Proof of Theorem 3:

Proof. From Proposition 2, we know that strict separation on any interval is not profit-maximizing for the Cartel, so attention can be restricted to round-bidding strategies on some grid $\mathcal{B}$.

The Cartel's problem with dynamic enforcement. Fix an incentive compatible round-bidding strategy $(\mathcal{C}, \mathcal{B}(\mathcal{C}))$ (see Proposition 2), and denote the expected payoff to a single compliant Cartel member,

$$\Lambda(\mathcal{C}) = \frac{1}{N} \sum_{\ell=1}^{L+1} H(c_{\ell-1}, c_\ell) \mathbb{E}[\lambda(c) | c_{\ell-1} \leq c < c_\ell]$$

and the Bayesian Nash payoffs in the strictly separating equilibrium,

$$\pi^* = \int_{\underline{c}}^{R} (1 - G(c_i))^{N-1} \lambda(c_i) dG(c_i) + \int_{R}^{\overline{c}} 0 dG(c_i).$$

Consider a perfect public monitoring strategy of the type:

- Phase 1: Bid according to $\mathcal{B}$. If the winning bid is on-$\mathcal{B}$, stay in Phase 1, otherwise move to Phase 2.
- Phase 2: Bid the non-collusive Bayesian Nash strategies

$$b^*(c_i) = c_i + \frac{\int_{c_i}^{R} (1 - G(z))^{N-1} dz}{(1 - G(c_i))^{N-1}}.$$

With probability ψ, stay in Phase 2, and with probability $1 - \psi$, return to Phase 1.

The Poisson punishment is useful because the system of equations that describes the firms' payoffs is continuous in $\psi \in [0, 1]$, while a discrete punishment period over $\mathbb{N}$ is not. Likewise, since the Cartel is programming its round-bidding strategy, it is advantageous to maximize over compact, convex sets like $[0, 1]$ that have convex images rather than a countably infinite set like $\mathbb{N}$.

Semi-continuity and continuity of deviation payoffs. Because round-bidding creates discontinuities in the probability of winning as a function of the type report $\hat{c}$, maximizers of the optimal deviation problem will typically not exist. Instead, we require that compliance delivers a better payoff than any available by deviating, and consequently, the supremum of the payoffs achievable by deviating.

Notice that since only the winning bid is revealed to the Cartel, deviations are only detected if the deviation wins the auction. It is logically possible that some type c_i has a strictly profitable

deviation, but the deviation in bidding is not detected in a given period. As a consequence of the One Shot Deviation principle, however, it suffices to consider a deviation from on-$\mathcal{B}$ bidding — which might be detected and trigger a deviation, or not — followed by a return to compliance. The intuition is that because we seek compliance, a deviation that is profitable and recurrent would already constitute a contradiction. The goal is to deter even a single deviation from compliance with a suitable punishment.

Let $\Delta(c_i, \mathcal{C})$ correspond to this supremum, the highest payoff a type c_i can get when faced with the grid $\mathcal{C}$ from an optimal one shot deviation:

$$\Delta(c_i, \mathcal{C}) = \sup_{b'} \Bigg\{ \text{pr}[\text{Lowest bid less than } R|b'] \left(b' - c_i + \frac{1}{1-\psi\delta} \left(\pi^* + \frac{(1-\psi)\delta}{1-\delta} \Lambda(\mathcal{C}) \right) \right) \\ + (1 - \text{pr}[\text{Lowest bid less than } R|b']) \left(0 + \frac{\delta}{1-\delta} \Lambda(\mathcal{C}) \right) \Bigg\}, \quad (8)$$

where the discounted value of the Poisson punishment is already calculated and included on the first line. The consideration of the optimal deviation b' is over the trade-off between winning and triggering the punishment, versus getting the chance to strategically deviate again.

Since $\mathcal{B}(\mathcal{C})$ is a finite subset of $[\underline{c}, R]$, there are a finite number of jump discontinuities in $\Delta(c_i, \mathcal{C})$. Define

$$p^d(c_i) = \limsup_{c \uparrow c_i} p(c) \quad \text{and} \quad b^d(c_i) = \limsup_{c \uparrow c_i} b(c),$$

where $\langle p, b \rangle$ correspond to the round-bidding strategy. These are everywhere continuous except at points on $\mathcal{C}$ and $\mathcal{B}$, which correspond to the supremum of the values the function takes in that neighborhood. Since the Revelation Principle guarantees that every game of incomplete information has an alternative representation as a Direct Mechanism, we can substitute the functions $\mu^d = \langle p^d, b^d \rangle$ into Equation (8) as a reporting game in which the deviator picks a type to report rather than a bid.

Substituting these definitions yields

$$\sup_{c'} \left\{ p^d(c') \left(b^d(c') - c_i + \frac{1}{1-\psi\delta} \left(\pi^* + \frac{(1-\psi)\delta}{1-\delta} \Lambda(\mathcal{C}) \right) \right) + (1 - p^d(c')) \left(0 + \frac{\delta}{1-\delta} \Lambda(\mathcal{C}) \right) \right\}$$

and $\Delta(c_i, \mathcal{C})$ is actually an upper semi-continuous function, since the payoffs at the discontinuities are now being adjusted up to the supremum of values that the function takes in an arbitrarily small neighborhood of the points of discontinuity. A maximizer is then guaranteed to exist by an extension of the Extreme Value Theorem to allow for upper semi-continuous objective functions, allowing us to write

$$\max_{c'} \left\{ p^d(c') \left(b^d(c') - c_i + \frac{1}{1-\psi\delta} \left(\pi^* + \frac{(1-\psi)\delta}{1-\delta} \Lambda(\mathcal{C}) \right) \right) + (1 - p^d(c')) \left(0 + \frac{\delta}{1-\delta} \Lambda(\mathcal{C}) \right)) \right\}.$$

With this adjustment, all type reports in the deviation mechanism μ^d except those on $\mathcal{C}$ are dominated. These are the only points at which the probability of winning changes, and it jumps up, so adopting an off-$\mathcal{C}$ report just reduces the payment the deviator might receive by reducing their bid but holding the probability of winning constant since their opponents' bids are only on $\mathcal{B}$: why further undercut yourself if you've already exploited the gains from undercutting the atoms? More formally, if a firm is already undercutting some b_ℓ, the probability $p^d(c')$ of winning does not vary on the interval $[c_{\ell-2}, c_{\ell-1})$, so there is no reason to further reduce one's bid below $b^d(c')$.

That means the maximization can be re-written as a simple linear programming problem in which the c_i type selects the probability of strategically undercutting each atom so that the payoff approaches the supremum:

$$\max_{x_1,\ldots,x_L} \sum_{\ell=1}^{L} x_\ell \left\{ p_\ell^d \left(b_\ell^d - c_i + \frac{1}{1-\psi\delta} \left(\pi^* + \frac{(1-\psi)\delta}{1-\delta}\Lambda(\mathcal{C}) \right) \right) + (1-p_\ell^d) \left(0 + \frac{\delta}{1-\delta}\Lambda(\mathcal{C}) \right) \right\} \quad (9)$$

subject to $0 \leq x_\ell \leq 1$ and $\sum_{\ell=1}^{L} x_\ell = 1$.

Now, the solution correspondence x^* will be an extreme, bang-bang solution. It will correspond to a unique ℓ, or the convex face of a fixed set of ℓ's which deliver the same expected payoff. Make a deterministic selection from the correspondence $\ell^*(c_i)$ that is right-continuous in c_i. Then the maximum payoff to deviating is

$$\begin{aligned} \Delta^*(c_i, \mathcal{C}) &= p_{\ell^*(c_i)}^d \left(b_{\ell^*(c_i)}^d - c_i + \frac{1}{1-\psi\delta} \left(\pi^* + \frac{(1-\psi)\delta}{1-\delta}\Lambda(\mathcal{C}) \right) \right) + (1 - p_{\ell^*}^d(c_i)) \left(0 + \frac{\delta}{1-\delta}\Lambda(\mathcal{C}) \right) \\ &= p_{\ell^*(c_i)}^d \left(b_{\ell^*(c_i)}^d - c_i + \frac{\pi^* - \psi\delta\Lambda(\mathcal{C})}{1-\psi\delta} \right) + \frac{\delta}{1-\delta}\Lambda(\mathcal{C}). \end{aligned}$$

The partition $\mathcal{C}$ picks out L bids $\mathcal{B}(\mathcal{C})$ along with the exogenous reserve price R, and only these types and bids are relevant to deciding the optimal deviation. Since those bids and probabilities vary continuously in $\mathcal{C}$, the payoff from the best undercutting bid likewise varies continuously in $\mathcal{C}$, and $\Delta(c_i, \mathcal{C})$ is a continuous function in the partition and type. Then $\Delta^*(c_i, \mathcal{C})$ varies continuously in $\mathcal{C}$, and the Theorem of the Maximum implies the value function also varies continuously in type c_i, as well as δ.

Enforcement with variable δ. A particular round-bidding strategy $(\mathcal{C}, \mathcal{B}(\mathcal{C}))$ is enforceable if, for all c_i,

$$p_\ell(b_\ell - c_i) + \frac{\delta}{1-\delta}\Lambda(\mathcal{C}) \geq \Delta^*(c_i, \mathcal{C}) = p_{\ell^*(c_i)}^d \left(b_{\ell^*(c_i)}^d - c_i + \frac{\pi^* - \psi\delta\Lambda(\mathcal{C})}{1-\psi\delta} \right) + \frac{\delta}{1-\delta}\Lambda(\mathcal{C}).$$

Re-arranging the above inequality yields

$$p^d_{\ell^*(c_i)} \frac{\psi\delta\Lambda(\mathcal{C}) - \pi^*}{1-\psi\delta} \geq p^d_{\ell^*(c_i)}(b^d_{\ell^*(c_i)} - c_i) - p_\ell(b_\ell - c_i). \tag{10}$$

The right-hand side is non-negative, since the flow payoff of the best deviation must be weakly greater than the flow payoff of honest reporting, or no deviation would be tempting. A simple upper bound on the right-hand side is $R - c_i$. On the left-hand side it must be the case that $\Lambda(\mathcal{C}) > \pi^*$, or the proposed collusive agreement would not be profitable. Further simplification yields

$$\frac{\psi\delta\Lambda(\mathcal{C}) - \pi^*}{1-\psi\delta} \geq \max_{c_i} \frac{p^d_{\ell^*(c_i)}(b^d_{\ell^*(c_i)} - c_i) - p_\ell(b_\ell - c_i)}{p^d_{\ell^*(c_i)}} \equiv D, \tag{11}$$

or

$$\psi\delta > \frac{\pi^* + D}{\Lambda(\mathcal{C}) + D}.$$

Since the right-hand side is strictly less than one, there is a locus of pairs $(\Psi^*, \Delta^*) \subset [0,1]^2$ for which any pair $(\psi, \delta) > (\Psi^*, \Delta^*)$ will satisfy the inequality. So for δ sufficiently close to 1, there is a punishment probability $\psi(\delta)$ for which the proposed strategies constitute a sub-game perfect Nash equilibrium and any incentive compatible $(\mathcal{C}, \mathcal{B}(\mathcal{C}))$ is enforceable for δ.

Enforcement with fixed δ. For a fixed $\mathcal{C}$ and δ, enforceability is characterized by

$$\Delta^*(c_i, \mathcal{C}, \psi) =$$
$$\max_{x_1,\ldots,x_L} \sum_{\ell=1}^{L} x_\ell \left\{ p^d_\ell \left(b^d_\ell - c_i + \frac{1}{1-\psi\delta}\left(\pi^* + \frac{(1-\psi)\delta}{1-\delta}\Lambda(\mathcal{C})\right)\right) + (1-p^d_\ell)\left(0 + \frac{\delta}{1-\delta}\Lambda(\mathcal{C})\right)\right\} \tag{12}$$

and the compliance constraint

$$0 \geq \Delta^*(c_i, \mathcal{C}, \psi) - \left(p_\ell(b_\ell - c_i) + \frac{\delta}{1-\delta}\Lambda(\mathcal{C})\right) \tag{13}$$

for all $c_i \in [\underline{c}, R]$.

Since $\Delta^*(c_i, \mathcal{C}, \psi)$ is continuous in its arguments and $c_i \in [\underline{c}, R]$, the compliance constraints can be subsumed into a single non-linear inequality,

$$0 \geq \rho(\mathcal{C}, \psi) \equiv \max_{c_i \in [\underline{c}, R]} \left\{ \Delta^*(c_i, \mathcal{C}) - \left(p_\ell(b_\ell - c_i) + \frac{\delta}{1-\delta}\Lambda(\mathcal{C})\right)\right\}.$$

Since $\rho(\mathcal{C}, \psi)$ is continuous in $\mathcal{C}$ and ψ and the inequality is weak, the set of grids $\mathcal{C} = \{c_1, ..., c_L\}$ that satisfy the constraint is a closed subset of the set of all grids, $\mathcal{G}$, and therefore compact.

Then the Cartel can solve the constrained maximization problem:

$$\max_{\psi,\mathcal{C}} \sum_{\ell=1}^{L+1} H(c_{\ell-1}, c_\ell)\mathbb{E}[\lambda(c)|c_{\ell-1} \leq c < c_\ell]$$

subject to

$$\begin{aligned} 0 &\geq \rho(\mathcal{C}, \psi) \\ \frac{H(c_{\ell-1}, c_\ell)}{N(G(c_\ell) - G(c_{\ell-1}))} &\geq \frac{H(c_\ell, c_{\ell+1})}{N(G(c_{\ell+1}) - G(c_\ell))}, \quad \ell = 1, ..., L. \end{aligned}$$

For a fixed δ and small L, there might be no feasible points, even if $\psi \to 1$ to maximize the severity of the punishment for deviating. If the Cartel members are too impatient, it must add additional bids to the grid to deter deviations, as in the case with a single reserve price bid R. However, the constraint set is never empty, since as L goes to infinity, the fully separating equilibrium is feasible and trivially enforceable. Therefore, a solution exists for the problem of maximizing Cartel profits for a fixed δ that generically involves multiple bid increments, since the above maximization problem entails maximization of a continuous function over a compact set.

Comparative statics with δ. Consider the correspondence Γ that maps discount factors $\delta \in (0, 1)$ into a set of incentive compatible and enforceable partitions of $\mathcal{C}$ that is a subset of the set of all grids, $\mathcal{G}$. The monotonicity constraints are a set of L non-negativity constraints that correspond to a non-empty subset of the set of all grids, $\mathcal{G}$, for all L. The subset of $\mathcal{G}$ on which the enforceability constraint is satisfied, however, might be empty for some L, as noted, but as $L \to \infty$, the separating equilibrium becomes a feasible point in $\mathcal{G}$ for which the enforceability constraint is satisfied, so there is always a solution to the Cartel's problem, and $\Gamma(\delta) \neq \varnothing$ for all $\delta \in (0, 1)$.

Recall Equation (11),

$$\frac{\psi\delta\Lambda(\mathcal{C}) - \pi^*}{1 - \psi\delta} \geq \max_{c_i} \frac{p^d_{\ell^*(c_i)}(b^d_{\ell^*(c_i)} - c_i) - p_\ell(b_\ell - c_i)}{p^d_{\ell^*(c_i)}}.$$

The right-hand side does not depend on δ or ψ, and the left-hand side is non-decreasing in δ and ψ. Then if $\delta' > \delta$ and the constraint is satisfied for $\mathcal{C}$ at δ, the constraint is also satisfied at $\delta' > \delta$. That implies that any $\mathcal{C}$ that is enforceable for δ is enforceable for $\delta' > \delta$, and $\Gamma(\delta') \supseteq \Gamma(\delta)$. Since the Cartel prefers to minimize separation, it will only add additional bids to $\mathcal{B}$ if the enforcement constraint binds, and if $\delta' > \delta$, $\mathcal{C}^*(\delta')$ will have weakly fewer elements than $\mathcal{C}^*(\delta)$. □

Proposition 4. *The probability of winning by making a report in $[c_{\ell-1}, c_\ell)$ or bidding b_ℓ is*

$$p(c_{\ell-1}, c_\ell) = \frac{H(c_{\ell-1}, c_\ell)}{N(G(c_\ell) - G(c_\ell))} = p_\ell$$

and

$$\frac{\partial H(c_{\ell-1}, c_\ell)}{\partial c_\ell} = N(1 - G(c_\ell))^{N-1} g(c_\ell).$$

Proof. Define

$$H(c_{\ell-1}, c_\ell) = \sum_{j=1}^{N} \frac{N!}{j!(N-j)!} (G(c_\ell) - G(c_{\ell-1}))^j (1 - G(c_\ell))^{N-j},$$

as appears in the proof of Proposition 2, as well as

$$H_k(c_{\ell-1}, c_\ell) = \sum_{j=k}^{N} \frac{N!}{j!(N-j)!} (G(c_\ell) - G(c_{\ell-1}))^j (1 - G(c_\ell))^{N-j},$$

so that $H_1(c_{\ell-1}, c_\ell) = H(c_{\ell-1}, c_\ell)$. These are similar to the distributions of order statistics, so some of the same recursion relations are satisfied, namely,

$$H_k(c_{\ell-1}, c_\ell) = H_{k-1}(c_{\ell-1}, c_\ell) - \frac{N!}{(k-1)!(N-k+1)!} (G(c_\ell) - G(c_{\ell-1}))^{k-1} (1 - G(c_\ell))^{N-k+1}. \quad (14)$$

Define $h_k(c_{\ell-1}, c_\ell) = \partial H_k(c_{\ell-1}, c_\ell)/\partial c_\ell$. We prove by induction that

$$h_k(c_{\ell-1}, c_\ell) = \frac{N!}{(k-1)!(N-k)!} (G(c_\ell) - G(c_{\ell-1}))^{k-1} (1 - G(c_\ell))^{N-k} g(c_\ell). \quad (15)$$

The base case is

$$H_N(c_{\ell-1}, c_\ell) = (G(c_\ell) - G(c_{\ell-1}))^N (1 - G(c_\ell))^0$$

with

$$h_N(c_{\ell-1}, c_\ell) = N(G(c_\ell) - G(c_{\ell-1}))^{N-1} g(c_\ell),$$

so the result is true for the $k = N$ case.

To complete the induction step, suppose that Equation (15) holds at the kth step:

$$h_k(c_{\ell-1}, c_\ell) = \frac{N!}{(k-1)!(N-k)!} (G(c_\ell) - G(c_{\ell-1}))^{k-1} (1 - G(c_\ell))^{N-k} g(c_\ell). \quad (16)$$

Differentiating the recurrence relation, Equation (14), yields:

$$\begin{aligned} \frac{\partial H_k(c_{\ell-1}, c_\ell)}{\partial c_\ell} &= \frac{\partial H_{k-1}(c_{\ell-1}, c_\ell)}{\partial c_\ell} \\ &\quad - \left[\frac{N!}{(k-1)!(N-k+1)!} (k-1)(G(c_\ell) - G(c_{\ell-1}))^{k-2} (1 - G(c_\ell))^{N-k+1} g(c_\ell) \right] \\ &\quad + \left[\frac{N!}{(k-1)!(N-k+1)!} (N-k)(G(c_\ell) - G(c_{\ell-1}))^{k-1} (1 - G(c_\ell))^{N-k} g(c_\ell) \right]. \end{aligned}$$

The left-hand side and third line cancel out by the induction hypothesis, leaving

$$h_{k-1}(c_{\ell-1}, c_\ell) = \frac{N!}{(((k-1)-1)!(N-(k-1))!}(G(c_\ell) - G(c_{\ell-1}))^{(k-1)-1}(1 - G(c_\ell))^{N-(k-1)}g(c_\ell)$$

which is the desired expression for h_{k-1}.

Finally, note that $H(c_{\ell-1}, c_\ell) = H_1(c_{\ell-1}, c_\ell) = H_1(c_{\ell-1}, c_{\ell-1}) + \int_{c_{\ell-1}}^{c_\ell} h_1(c_{\ell-1}, z)dz$. Then, by the Fundamental Theorem of Calculus,

$$\begin{aligned} H(c_{\ell-1}, c_\ell) &= 0 + \int_{c_{\ell-1}}^{c_\ell} \frac{N!}{0!(N-1)!}(G(z) - G(c_{\ell-1}))^0(1 - G(z))^{N-1}g(z)dz \\ &= \int_{c_{\ell-1}}^{c_\ell} N(1 - G(z))^{N-1}g(z)dz, \end{aligned}$$

as in Proposition 2. □