A sketch of the model

We are interested in how households use resources, and how their resource use patterns are influenced by their own situations, village-level conditions such as social networks that provide information about markets for labor or agricultural products, and forces from the wider economy transmitted through labor markets, product prices, transport costs, and so on. As a starting point we aim to build a very simple model, capturing the main stylized elements of the household decision problem. Later, more complexity can be built into the model in order to improve its match with "real-world" conditions.

Suppose that households begin with endowments of upland, lowland, and unskilled labor, and that they aim to maximize the present value of a stream of net income from these. They can accomplish this either by direct production (agriculture) or by a combination of production and sale (of agric. products and of unskilled labor thru migration), or by any combination of these plus investments in land or labor quality (including negative investments, i.e. land degradation).

We can think of a sequential decision process for allocation of land and labor resources by farm households, as follows:

A. How much labor should be sent to seek non-farm employment? (this depends on non-farm labor opportunities and wages as well as labor productivity on-farm, and determines farm labor availability).

B. How much upland should be cultivated? (Depends on lowland endowments, labor availability, technology, prices, etc., and determines share of upland left fallow and allocation of labor between upland and lowland fields). The outcome also influences upland land degradation processes.

C. Can total household income (farm and non-farm sources) be improved by reallocating land and labor resources?

Yes

No

Stop

Return to A
A semi-formal treatment of the model

In this section we provide an intuitive exposition of the model and link it to existing literature. We outline major assumptions and methodologies, and introduce some notation.

Decision A: how much household labor should be sent to work at non-farm jobs?

At the optimum, the marginal contributions to household income from farm and on-farm labor should be equal. If they were not, then income could be increased by reallocating some labor from less rewarding to more rewarding activities. The non-farm labor market provides a benchmark. Let $w$ be the daily wage of non-farm labor (e.g. in an urban construction job). After Harris and Todaro (1970) we define the expected non-farm wage as $\rho w$, where $\rho < 1$ is the probability that having migrated, an individual will actually obtain a job in a given period. Then the optimal allocation of labor between farm and non-farm work requires that the value marginal product of farm labor ($VMP(L^f)$) be equal to the expected non-farm wage, or:

$$VMP(L^f) = \rho w.$$  

Out-migration from a household will occur until this condition is met, with each increment in out-migration reducing the labor-land ratio and thus raising the productivity of the remaining farm labor (of course, in-migration is also possible if $VMP(L^f) > \rho w$ initially). The migration decision thus determines how much labor will be allocated to farm activities as a whole.

Note that our construction assumes that the returns to farm labor are not determined solely in a rural labor market. It is reasonable to assume that the return to household labor used on the farm includes a component due to specific skills or managerial inputs (detailed knowledge of the land and soil conditions, for example), and thus that the rural wage is not a good measure of $VMP(L^f)$.

Additional comments (to be explored in more detail later)

1. Technical change in agriculture, or the adoption of a different cropping pattern, or the degradation of land quality over time all affect the migration decision by raising or lowering the productivity of farm labor. One example is the adoption of a high-valued upland crop such as cabbage. Under some conditions this land use shift raises the productivity of labor to the point that out-migration may be halted or even reversed.

2. Information about non-farm employment opportunities is costly for individuals to acquire. However, this cost may be offset by a number of factors, including communications with previous migrants from the household or village. We can capture the contribution of such social networks to the labor allocation decision by introducing an information cost parameter $\alpha < 1$. Higher values of $\alpha$ reflect better social networks, and a value of $\alpha = 1$ indicates perfect information about non-farm employment conditions.
For any household, the expected non-farm wage is higher, the more is known about employment opportunities. Thus, amending (1):

\[(1') \quad \text{VMP}(L^f) = \alpha \rho w.\]

In a household or village where \(\alpha\) is low, the expected non-farm wage is lower than if \(\alpha\) were closer to 1. When \(\alpha\) is low, less migration will occur (\(L^f\) will be higher for given household size).

3. We could make a similar argument about the role of social networks in providing information about agricultural product markets. Then the expected farm gate price of a product to a given household \(H_h\) would equal its market price times a market information cost parameter \(\beta_h \leq 1:\)

\[p_h = \beta_h p. \quad h = 1, \ldots, N.\]

4. The Harris-Todaro labor market model incorporates the idea that migration will not fully clear the labor market; some urban unemployment will always exist. The willingness of workers to remain in urban areas even without work can be interpreted as an irreversibility in the labor market, with implications for agricultural resource allocation (Coxhead and Jiraporn 1998).

**Decision B: How much upland land should be cultivated?**

Once \(L^f\) has been decided, we can evaluate on-farm production and resource allocation decisions separately—assuming, for the moment, no changes in external (market) conditions or in the values of soil quality state variables (since these changes will alter the optimal migration decision).

To see how farm decisions are made, consider a very simple example. Suppose that production takes place on lowland and upland plots, that land and labor are the only inputs used, that total upland and lowland areas are fixed (the upland area cultivated may be less than the total if some land is left fallow), and that only one crop is grown on each type of land. Upland and lowland production take place according to the production functions \(f(N^i, L^i, Z^i)\), where \(Z^i\) is a vector of non-factor inputs (technology, \ldots) and \(i = u, \ell\) for upland and lowland. With constant returns to scale, the amount of labor used in lowland production will be fixed by \((1')\), i.e.,

\[(2) \quad pf'_2 = \alpha \rho w.\]

where \(f'_2\) denotes the partial derivative of the lowland production function with respect to the second element, i.e., the marginal product of lowland labor. Once \(L^f\) has been decided in this way, any remaining labor will be allocated to upland production. Since the optimum also requires that \(pf'_2 = f'^u_2\), upland will be used up to the point where this
condition is satisfied. In other words, the share of upland that is fallowed, \( s \), will be
determined by the availability of labor to work upland plots. For given values of \( w, p, N_i, L, Z_i, \alpha, \beta, \) and \( \rho \), decisions A and B result in optimal household allocations of labor to
farm and non-farm activities and between upland and lowland plots, and the amount of
upland to be fallowed, in each period.

The simplicity of this example provides clear and intuitive results, based entirely on two
conditions: that the optimal allocation of household labor is equates its marginal
contribution across all activities, and that all labor not employed off-farm or on lowland
plots is employed on upland plots.

Finally, in this simple version with a fixed (physical) upland area and a single upland
crop, the rate of upland soil quality degradation will be determined by the intensity of
cultivation, i.e. by the values of \( s \). The more production that takes place, the faster the
rate of degradation, i.e. \( \frac{\partial Z_u}{\partial s} < 0 \), where \( Z_u \) is defined as the effective (quality-adjusted)
upland area. Future decisions about land and labor allocation will be influenced by
current decisions on \( s \); a higher value of \( s \) this year may mean different labor allocation
and fallow decisions next year.

**Additional comments**

1. Even this simple example allows us to examine ways in which external forces and
social networks influence resource allocation and environmental change in uplands.
Policies enter households' decisions through \( w, p, \) and \( \rho \); social networks at the village
level determine the values of \( \alpha \) and \( \beta \).

2. Because current decisions affect future upland quality, there is an irreversibility in land
quality that could potentially drive households into "optimal" degradation of land— or
cause them to abandon upland production altogether. This irreversibility may interact
with that already identified in the non-farm labor market.

3. Households that migrate might choose to lease out some land to others, and rents paid
on this would then form part of current income. Institutional features of such
arrangements, such as the power of lessors to abrogate contracts when non-farm
employment contracts, are potentially interesting subjects for study.

4. Other forms of investment, notably education or other "human capital" investments,
are certain to be of interest and could be incorporated in a future version of the model.

**Decision C: Can total household income be improved by reallocating resources?**

In each period (each season, perhaps) the household re-evaluates its resource allocation
decisions in the light of changes in land and labor endowments, upland quality, and the
market, policy and social network parameters identified above. Because the goal is to
maximize the present value of income, and because decisions are linked across periods
by upland quality changes and labor market irreversibilities, the solution to the model is
best obtained by dynamic programming. In this algorithm the household's problem is represented as the optimization of a value function over two time periods: the present, and all future periods discounted back to present values. Because future decisions depend on present allocations, the solution to a dynamic programming problem is achieved by backward recursion (Conrad and Clark; many others). In such a solution, the current allocation of resources is always the optimal allocation, for given values of exogenous variables; the question posed as Decision C is always answered in the negative.
1. A model with migration and land quality degradation

Let \( N, L, \) and \( Y \) be current period values of land, labor, and ag. output. Each is a vector (denoted by underscore). \( N = \{N_u, N_l\} \) (upland and lowland); \( L = \{L_u, L_l, L_m\} \) (upland, lowland and migratory (non-farm)); \( Y = \{Y_u, Y_l\} \) (upland and lowland). Upland and lowland production take place according to \( f(N_i, L_i, Z_i) \), where \( Z_i \) is a vector of non-factor inputs (fertilizer, technology, …). Migration is costly and irreversible in the short run, so labor decisions are made sequentially: migrate or stay home, then work on upland or lowland fields. Moreover, migration itself involves an investment of time and effort that reduces the "real" quantity of migrant labor by a factor \( (1-\lambda) < 1 \). Thus \( L - (1-\lambda)L_m = L_u + L_l \) is the household labor constraint.

When labor migrates, some land may be let out to others in the village at a net annual rental rate \( \mathbf{R} = \{R_u, R_l\} \). In addition, upland land may be cultivated or left fallow in any year. The share of upland that is cultivated, \( s \), reflects this choice. Finally, land is measured in quality-adjusted units. While lowland may safely be assumed to be of about uniform and constant quality, the quality of upland is a function of the length of fallow periods and perhaps of the types of crops grown on it. Thus the (effective) upland land endowment is a state variable, whose value in any period is given by the physical upland area and its average quality. Land degrades according to the function \( N = N(Y^{-1}), N' < 0, N'' > 0 \), indicating that production degrades agricultural land over time.

Decision-makers face a number of "givens", including the prices of agricultural outputs and inputs, non-farm wages, transport and transactions costs, and rental rates for leased land of a certain quality. Some other variables may depend on characteristics of the village: one example is the cost of acquiring information about migratory employment opportunities.

In a two period setting, households maximize \( R = R_1 + \phi R_2 \), where \( \phi \) is a discount factor and

\[
R = PY(N,L) + WL(f) + \beta WL(n) - C(W) + \phi R_2
\]

\[
R_2 = PY(N,L) + WL(f) + \beta WL(n) - C(W) \quad \text{(for future period values)}
\]

Task: Set this up as a maximization problem with appropriate constraints (technology, land and labor endowments, and land quality state equation). Show that in equilibrium, returns to labor are equalized across occupations, and that exogenous shocks such as changes in \( \alpha, \beta, \) and \( \lambda \) can cause changes in the return to land, thus its productivity at the margin, thus the value of investments in future land productivity.

Useful sources: Chavas 1994 (risk); Vousden 1990 (2-period model with adjustment costs); Frenkel and Razin 1987 (2-period model of production, investment and debt); Manuelli, EC712 notes (dynamic optimization).
The model.
Max \((L^M, L^I, s)\) \(\int_0^\infty U(\Pi) e^{-\rho t} dt\),

Where current-period income is given by:

\[ \Pi = p_h f^I(N_u, L^I, Z^u) + f^u(sN^u, L - L^M - L^I, Z^u) + (\rho w - a)L^M - cD \]

subject to

\[ \frac{dN^u}{dt} = D - b(s)sN^u \quad b'(s) > 0, b''(s) > 0. \]
\[ 0 \leq s \leq 1 \quad \text{(share of upland cultivated)} \]
\[ N^u \geq A \quad \text{(total upland area)} \]

and boundary conditions.

Form the current-period Hamiltonian, \(H\):

\[ H = U(\Pi) + \lambda(D - b(s)sN^u) + \omega_1 s + \omega_2 (1-s) + \omega_3 (A - N^u) \]

FONC:

\[ \frac{\partial H}{\partial L^M} = -U'(\Pi)[f^u_2 - (\rho w - a)] = 0 \]
\[ \frac{\partial H}{\partial L^I} = U'(\Pi)[p_h f^I_2 - f^u_2] = 0 \]
\[ \frac{\partial H}{\partial s} = U'(\Pi)f^u_1 - \lambda[b(s)N^u + b'(s)sN^u] + \omega_1 - \omega_2 = 0 \]
\[ \frac{\partial H}{\partial D} = -U'(\Pi)c + \lambda = 0 \]
\[ \lambda^* = -\frac{\partial H}{\partial N^u} + \lambda r \]
\[ = -U'(\Pi)[f^u_1 s - \omega_1] + \lambda b(s)s + \lambda r \]

Assume interior solution? Not necessarily, since \(0 \leq L^M \leq L\), and same for \(L^I\).

Equilibrium labor allocation is

\[ (\rho w - a) = p_h f^I_2 = f^u_2 \], so migration occurs if \(w - (p_h f^I_2 + a)/\rho > 0\).

Other things equal, less migration will occur if urban wages fall, farm prices rise, if labor productivity on farm rises, if information costs rise, or if urban unemployment rises (since \( \rho < 1 \)).

To think about
- Adjustment costs
- Anticipated and unanticipated shocks, and responses
- Uncertainty
- Crop diversification in uplands