THE ECONOMICS OF GROWTH

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10 Stages of Growth

10.1 Introduction

Countries tend to go through various stages of growth, in which the rate of growth, the sectoral composition of growth, and also the main driving forces of growth change. This chapter studies how growth evolves from one stage to another, in three different dimensions. First we study the transition from Malthusian stagnation to modern economic growth, associated with the industrial revolution. Next we study the transition from growth based on capital accumulation to more innovation-based growth. Finally, we study the transition from manufacturing to a service economy that has been taking place for the past century in leading economies.

10.2 From Stagnation to Growth

Sustained long-term economic growth at a positive rate is a fairly recent phenomenon in human history, most of it having occurred in the last 200 years. According to Maddison’s (2001) estimates, per capita GDP in the world economy was no higher in the year 1000 than in the year 1, and only 53 percent higher in 1820 than in 1000, implying an average annual growth rate of only one-nineteenth of 1 percent over the latter 820-year period. Some time around 1820, the world growth rate started to rise, averaging just over one-half of 1 percent per year from 1820 to 1870, and peaking during what Maddison calls the “golden age,” the period from 1950 to 1973, when it averaged 2.93 percent per year. By 2000 world per capita GDP had risen to more than $8\frac{1}{2}$ times its 1820 value.

There is now a substantial literature on unified growth theory, which attempts to build a growth model that applies not only to the modern era of sustained growth but also to the much longer period before the industrial revolution, when growth was negligible. A comprehensive survey of this literature can be found in Galor (2005). The following account is built on Hansen and Prescott (2002) and Ashraf and Galor (2008).

10.2.1 Malthusian Stagnation

10.2.1.1 Population and Per Capita Income

The basic ideas of unified growth theory come from Malthus (1798), who argued that long-run growth in living standards was impossible. The problem, according to Malthus, came from population growth and diminishing returns to labor.
Specifically, if per capita income were to rise substantially, then people would survive longer and have larger families, which would raise the population. But as population rose, per capita income would fall because more people would be working with a fixed amount of land. In the end, per capita income would fall back to where it was in the first place.

To see how this works in greater detail, consider an economy that is entirely agrarian, where aggregate output $Y$ is just the output $Y_a$ produced in agriculture using labor $L_a$ and land $X$, according to the production function

$$Y_a = AX^\beta L_a^{1-\beta}, \quad 0 < \beta < 1$$

(10.1)

where for now we take the productivity parameter $A$ as given. For simplicity of notation, suppose that $X = 1$. Suppose that everyone in the population supplies one unit of labor, so $L_a$ is equal to the population size $L$. Then per capita income will be

$$y = Y/L = AL^{-\beta}$$

(10.2)

which goes up when productivity $A$ increases but falls when population $L$ increases.

The reason why population growth reduces per capita income is diminishing returns. That is, although the production function (10.1) exhibits constant returns to scale, in the sense that an equiproportional increase in both factors, land and labor, would have a proportional effect on output, leaving per capita income $y$ unchanged, nevertheless an increase in population will have a less than proportional effect on output, because land is a fixed factor that cannot be increased along with labor.

Now, suppose that higher standards of living translate into a higher rate of population growth. More specifically, the growth rate of population depends on per capita income according to the function $n$:

$$\dot{L}/L = n(y)$$

(10.3)

where

$$n'(y) > 0$$

Suppose also that population growth has a fixed upper limit $n^{\text{max}} > 0$ and that when people are poor enough it becomes negative:

$$n(y) \to n^{\text{max}} > 0 \quad \text{as} \quad y \to \infty \quad \text{and}$$
Then, as figure 10.1 shows, there will a unique level $y^*$ of income per capita at which population growth is just equal to zero:

$$n(y^*) = 0$$  \hspace{1cm} (10.4)$$

Using equation (10.2) to substitute for $y$ in the population-growth equation (10.3) yields

$$\dot{L}/L = n(AL^{-\beta})$$

Given the level $A$ of productivity and some initial level $L_0$ of population, this differential equation can be solved for the entire future time path of population. It implies that there is a unique steady-state level of population $L^*$ such that population growth will equal zero:

$$n(A(L^*)^{-\beta}) = 0$$  \hspace{1cm} (10.5)$$

Combining equations (10.4) and (10.5) we see that

$$L^* = (A/y^*)^{1/\beta}$$  \hspace{1cm} (10.6)$$
Population will stabilize at the level \( L^* \) in the long run, no matter where it starts. That is, if initially \( L \) were to exceed \( L^* \), then per capita income \( y = AL^{-\beta} \) would be less than \( y^* \) and therefore \( L \) would fall back toward \( L^* \). As long as \( L > L^* \), the same force would be at work, so that \( L \) would in fact converge to \( L^* \) from above. By the same reasoning, if \( L \) were to start below \( L^* \), then it would converge to \( L^* \) from below.

Now the fact that population converges to \( L^* \) in the long run implies that per capita income, which depends on population according to equation (10.2), will converge to the corresponding value \( y^* \). In other words, per capita income will ultimately stagnate, not growing beyond the level \( y^* \).

### 10.2.1.2 The Effects of a Productivity Increase

The Malthusian stagnation result was demonstrated on the assumption of a given level of total factor productivity. Could productivity growth save people from stagnation? As it turns out this is not so easy.

**Level Increase**

Consider first the case of a once-over increase in productivity to the new level \( A' > A \). If the economy were initially at its steady-state level with

\[ L = L^* = \left( A/y^* \right)^{1/\beta} \]

and

\[ Y/L = y^* \]

then the productivity increase would at first raise per capita income, from \( y^* \) to \( A'L^{-\beta} > AL^{-\beta} = y^* \)

But this short-run increase in per capita income would start the Malthusian process at work again. Because per capita income is now above the level \( y^* \), the population will start to rise, and this increase will start to bring per capita income back down again. This process will continue until per capita income has fallen back to the level that just eliminates population growth—that is, until it has fallen back to its initial level \( y^* \).

Note that although the increase in productivity will have no lasting effect on income per capita, it will have a lasting effect on the size of population. Indeed, it is because of this lasting effect on population size that there is no lasting effect on per capita income. More formally, equation (10.6) indicates that the steady-
state population size $L^*$ is an increasing function of productivity. Galor (2005) reviews a great deal of historical evidence confirming this prediction of Malthusian theory, evidence that more technologically advanced countries and regions, as well as those with a higher quality of land, did not have a much higher standard of living but did have a much higher population density, as compared to those with less advanced technology and inferior land quality.

**Steady Productivity Growth**

The analysis has so far taken the productivity parameter as being an exogenous constant, perhaps subject to occasional increases but not steady growth. However, there was considerable growth in the world’s stock of technological knowledge prior to the industrial revolution, not as a result of large-scale organized R&D of the sort we are familiar with in advanced countries nowadays but as the result of a combination of the cumulative buildup of lessons learned from experience, creative experimentation by scholars, the need to deal with environmental crises, and the natural human propensity to tinker with things. It is generally agreed, however, that technological progress was a slow process in the Malthusian era. Although there were probably periods in which great discoveries were made (fire, the wheel, the windmill, ...), there were also long periods of relative technological stagnation.

Suppose we take this factor into account by allowing for some steady exogenous drift in the productivity parameter $A$. Then our conclusions would be not quite as pessimistic as under Malthusian theory but almost. Specifically, suppose that

$\dot{A}/A = g$, a positive constant

Then according to equation (10.2) the growth rate of per capita income will be

$\dot{y}/y = \dot{A}/A - \beta \dot{L}/L = g - \beta n(y)$

This is a stable differential equation that now converges not to $y^*$ but to a somewhat higher level $\tilde{y}$, the level that satisfies the stationary condition

$n(\tilde{y}) = g/\beta$ (10.8)

1. It follows immediately from equation (10.2) that

$\ln y = \ln A - \beta \ln L$

Taking derivatives of both sides of this equation with respect to time produces equation (10.7).
If $g$ is close to zero, then $\tilde{y}$ will be close to the level $y^*$ of Malthusian theory. As long as $^2 g < \beta n^{\text{max}}$, which undoubtedly was true for almost all of human history, technological progress will still not produce sustained growth in per capita income. Instead, it will just affect the level at which stagnation occurs.

**10.2.2 The Transition to Growth**

**10.2.2.1 Agriculture and Manufacturing**

The long period of stagnation ended around the time of the industrial revolution, when some countries started reallocating resources away from agriculture and toward manufacturing. To see how this shift might happen, suppose that there is a latent manufacturing technology which has not yet been economically viable, but which, as we shall see, will become viable once technology has progressed enough.

Manufacturing output $Y_m$ is produced by labor alone, according to

$$Y_m = AL_m$$

(10.9)

The productivity parameter $A$ is assumed for simplicity to be the same as in agriculture. Total output is now

$$Y = Y_a + Y_m$$

and the two sectors will compete for the given labor supply $L$

$$L_a + L_m = L$$

(10.10)

The two production functions (10.9) and (10.1) are the same except that one uses land while the other does not. So whether the manufacturing technology is used or not will depend on how much land there is relative to labor. When population is very small, there is a lot of land for each person to work with, so no one will produce using the manufacturing technology because it does not take advantage of this abundant factor. But if population were to rise beyond a critical level, then it would be profitable to activate the manufacturing technology in order to escape the land constraint.

**10.2.2.2 Wages and Industrialization**

To see how this process works, suppose the economy is in a Malthusian equilibrium with a rate of technological progress $g$, which we can suppose is very close

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2. If $g > \beta n^{\text{max}}$, then population growth will not be able to undo the effect of productivity increases, because equation (10.7) indicates that the growth rate of per capita income cannot fall below $g - \beta n^{\text{max}} > 0$. 

to zero, so per capita income has stagnated at the level $\tilde{\gamma}$. According to equation (10.8), population will be growing at the rate $g/\beta$. The real wage rate will equal the marginal product of labor

$$\tilde{w} = (1 - \beta)AL^{-\beta} = (1 - \beta)\tilde{\gamma}$$

which will be constant over time.

Someone wanting to hire labor to produce output using the manufacturing technology would earn a profit equal to

$$\Pi_m = Y_m - \tilde{w}L_m = (A - \tilde{w})L_m$$

So as long as the stock of knowledge is relatively low, $A$ will be less than $\tilde{w}$, and it will not be profitable to use the manufacturing technology. But if $A$ continues to grow, then at some point it will surpass $\tilde{w}$, which remains constant in the Malthusian equilibrium. At this point it becomes profitable to activate the manufacturing sector, and industrialization begins.

Once manufacturing has started, free entry guarantees that the profit rate $\Pi_m$ will be zero, so the wage rate will equal the productivity parameter

$$w = A$$

The agricultural sector will also have to pay that wage, so its profit (excluding the rent to the fixed factor land) will be

$$\Pi_a = AL_a^{-\beta} - wL_a = A(L_a^{-\beta} - L_a)$$

Employment in the agricultural sector $L_a$ will then be chosen to maximize profit, and the first-order condition determining this profit-maximizing choice is

$$\frac{\partial \Pi_a}{\partial L_a} = A[(1 - \beta)L_a^{-\beta} - 1] = 0$$

which solves for the constant value

$$L_a = (1 - \beta)^{1/\beta}$$

From this point on, while agricultural labor remains constant, manufacturing labor will equal

$$L_m = L - L_a$$

which will continue to rise over time, since $L_a$ is constant and the total labor supply $L$ will continue to grow, starting at the Malthusian rate $n(\tilde{\gamma})$. 
10.2.2.3 Sustainable Growth

Not only will population continue to grow, but it will accelerate, and the rate of economic growth $\dot{y}/y$ will rise from its previous rate of zero to a new steady-state value equal to the rate of technological progress $g$. To see how this process works, notice that income per capita is now

$$\frac{y}{L} = \frac{A(L - L_a)}{L} + \frac{AL^{1-\beta}}{L}$$

Over time the agricultural share will approach zero, since $L_a$ is constant and $L$ becomes infinitely large. Income per capita will thus become approximately equal to $A$ and therefore will grow without bound, approaching the growth rate $g$ of productivity. And the population growth rate will approach its upper limit $n^{\text{max}}$.

This transition from stagnation to growth takes place because the gradual accretion of knowledge has allowed people to escape from the limitations of land, especially from the diminishing marginal returns to labor applied to fixed land which kept wages and per capita income from taking off. Once freed from land, people are no longer subject to diminishing returns, and there is nothing to choke off growth in per capita income.

10.2.3 Commentary

The preceding analysis focuses on industrialization as the key to making the transition from stagnation to growth. Other theories focus on human-capital accumulation. For example, Galor and Weil (2000) construct an overlapping-generations model in which people derive utility from consumption and from the quantity and quality of children, where “quality” means the amount of human capital invested in the child.

In their model, the rate of technological progress is assumed to depend on the size of population, through a scale effect, and on the amount of human capital per person. The scale effect triggers the escape from Malthusian stagnation by gradually raising the rate of technological progress. At first the resulting increase in productivity growth leads to a population explosion, but at some point there is a demographic transition that brings population growth under control without per capita income having to fall. This demographic transition comes from a tilting of the quality-quantity trade-off; that is, a faster rate of technological progress raises the rate of return to human-capital investment because human capital is most valuable during periods of rapid technological change.3 People respond to

the increased skill premium by substituting more investment in quality for increased quantity of children.

The Galor-Weil theory has the advantage of predicting not only the industrial revolution but also the demographic revolution that took place around the same time; population growth did indeed start to fall. However, Mokyr and Voth (2006) have questioned this line of analysis on the grounds that there is little evidence that Europe was in a Malthusian trap in 1700. Population and per capita income had been rising together in many regions even though Malthusian theory implies that a rise in population should depress per capita income. Likewise, the population in England was roughly constant between 1700 and 1750, long before the industrial revolution that was supposed to have been triggered by the scale effect of a large population. Nor can historians find evidence of the skill premium that the human capital theory predicts should have induced the tilting of the quality-quantity trade-off that caused the demographic transition.

Mokyr and Voth suggest alternative factors that might have kept output from taking off before the industrial revolution. One is the development of a scientific culture among a critical mass of craftsmen and business people, the factor stressed by Howitt and Mayer-Foulkes (2005). Another is the set of institutions that protect property rights. Suppose, for example, that anyone could steal another person’s capital at a cost of $\sigma$. Then no one would accumulate capital beyond the level $K = \sigma$ because to do so would result in losing everything to a thief. In an AK model, that limit on capital accumulation would bring growth to an end until some institutional change raised the cost $\sigma$ of stealing someone else’s property.

The evidence will probably never settle the dispute among these competing theories of the transition from stagnation to growth. For, as Mokyr and Voth admit, there was only one industrial revolution, and one data point is just not enough. As a result, the Mokyr-Voth critique of Malthusian theories is not decisive, especially when we take into account that there is two-way causation between population and income, and that there may be long lags involved in these causal relationships. For example, the fact that population and per capita income were rising simultaneously in many regions of Europe before the industrial revolution might have been the joint effect of a gradual improvement in technology; perhaps the rise in per capita income would have been reversed eventually if the industrial revolution had not taken place.
10.3 From Capital Accumulation to Innovation

When growth first starts, it is driven by capital accumulation, which accommodates the gradual increase in knowledge without being subject to the diminishing returns that accrue to land. But then innovation takes over more and more. Figure 10.2 from Ha and Howitt (2006) shows that over the second half of the 20th century the contribution of capital accumulation (physical and human) to growth of per capita income has fallen from about one percentage point per year to about one half. This finding has important consequences for the current debate over the sources of growth (see chapter 5). As time passes, it seems that growth is becoming more and more knowledge based, driven less by the accumulation of human and physical capital and more by innovation. This section, drawn mainly from Ha (2002), analyzes the reasons for this particular transformation.

10.3.1 Human-Capital Accumulation

Human-capital accumulation in the form of education cannot contribute indefinitely to growth because, unlike physical capital, it is embodied in people, and people are mortal. When they die, so does their human capital. Consider, for example, a stationary population in which people devote the fraction \( u \) of their time to education. Over time the education of young people will raise years of
schooling per person $s$ at the rate $u$. But if the fraction $\delta$ of people per year die, and those who die have on average $s$ years of schooling, then these deaths will reduce $s$ at the rate $\delta s$. The net effect of education and death is to make average years of schooling change at the rate

$$\dot{s} = u - \delta s$$

which means that average years of schooling will go to a steady-state value (where $\dot{s} = 0$) of

$$s^* = \frac{u}{\delta}$$

According to Mincer (1974), human capital per person $h$ can be represented as an exponentially increasing function of years of schooling

$$h = e^{\theta s}$$

which means that human capital per person will approach a steady-state level

$$h^* = e^{\theta u / \delta}$$

Increasing the fraction of time spent on education will raise this steady-state level of human capital per person, but it cannot result in a permanent increase in the growth rate of human capital per person. Therefore, at some point the direct contribution of human-capital accumulation to economic growth must come to an end.

This is not to say, however, that human capital eventually ceases to be important for economic growth. On the contrary, because R&D is a human-capital-intensive activity, therefore as growth becomes more and more dependent on innovation, it also becomes more and more dependent on the level (and quality!) of educational attainment per person.

### 10.3.2 Physical-Capital Accumulation

In contrast to human capital, there is no limit to how much physical capital a person can accumulate. Yet even physical-capital accumulation is becoming less important than innovation as a source of growth. To understand this trend we reproduce the hybrid neoclassical-Schumpeterian model that includes both capital accumulation and innovation, which we developed in chapter 5, and we analyze its dynamic evolution.

The basic idea of this analysis is that as an economy develops following industrialization, it goes through a process of “capital deepening,” in which the emerging capital-intensive manufacturing technology becomes equipped. As we saw
in chapter 5, a higher productivity-adjusted capital stock creates a scale effect that encourages more innovation, because a successful innovator gets a bigger reward in a wealthier economy. But as in the neoclassical model, the higher productivity-adjusted capital stock lowers the marginal product of capital, which weakens the impact of capital accumulation on growth. So over time the contribution of capital accumulation to growth decreases while the contribution of innovation increases.

10.3.2.1 The Hybrid Model

The final good in this hybrid model is produced under perfect competition according to the production function

\[ Y_L = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} dt, \quad 0 < \alpha < 1 \]

where each \( x_{it} \) is the flow of intermediate input \( i \). For simplicity, we set \( L = 1 \). Each intermediate input is produced according to the production function

\[ x_{it} = K_{it} \]

where \( K_{it} \) is the amount of capital used as input. In equilibrium, each sector will produce at a level \( x_{it} \) that is proportional to the productivity parameter \( A_{it} \). When we put these equilibrium values into the production function, we get a neoclassical aggregate production function of the form

\[ Y_L = (A_L L)^{1-\alpha} K_t^\alpha \]

where \( K_t \) is the aggregate capital stock and the labor-augmenting productivity variable \( A_t \) is just the average across all sectors of the individual productivity parameters: \( A_t = \int_0^1 A_{it} dt \).

As we saw in chapter 5, there are three fundamental equations driving the economy. First, there is the neoclassical law of motion for the capital stock:

\[ K_{t+1} - K_t = sY_t - \delta K_t \]  \hspace{1cm} (K)

where \( s \) is the saving rate and \( \delta \) is the depreciation rate.

Next, there is the productivity-growth equation:

\[ \frac{A_{t+1} - A_t}{A_t} = g_t = (\gamma - 1)\phi(n_t) \]  \hspace{1cm} (G)

where \( \gamma \) is the size of innovations, \( n_t \) is the productivity-adjusted research expenditure in each sector, and \( \phi(n_t) \) is the innovation rate.
The final equation is the research-arbitrage condition:

$$\phi'(n_t) \bar{\pi}(\kappa_t) = 1$$  \hspace{0.5cm} (R)

where $\kappa_t = K_t/A_t$ is the productivity-adjusted capital stock and $\bar{\pi}(\kappa_t)$ is each monopolist’s productivity-adjusted profit, which is increasing in $\kappa_t$. The research-arbitrage equation makes $n_t$ an increasing function of $\kappa_t$, so the productivity-growth rate determined by equation ($G$) is also an increasing function of $\kappa_t$.

Each period, given the historically predetermined value of $\kappa_t$, the research-arbitrage equation (R) determines the research intensity $n_t$; then $K_t$ and $A_t$ change according to equations (K) and (G), which then determine $\kappa_{t+1}$, and the process is repeated next period, ad infinitum.

The steady state of the model is a situation in which all three variables $g$, $n$, and $\kappa$ are constant over time. The steady-state conditions are

$$g = (\gamma - 1)\phi(n)$$

$$\phi'(n) \bar{\pi}(\kappa) = 1$$

$$s\kappa^{\alpha-1} = \delta + g$$

The first two are just steady-state versions of equations ($G$) and (R), while the last one is the familiar neoclassical condition that requires the growth rate of capital per efficiency unit of labor (that is, the growth rate of $\kappa$) to be constant.

Now consider an economy that is at an early stage of growth, with an adjusted capital stock $k_0$ far below its steady-state value. In that economy, capital accumulation is very rapid while productivity growth is slow. But as $\kappa_t$ rises up to its steady-state value, the contribution of capital accumulation to growth will slow down while innovation and productivity growth speed up.

More specifically, we can write the aggregate production function as

$$Y_t = A_t \kappa_t^\alpha$$

so if $G_t$ is the growth rate of output, then we can express the growth factor $1 + G_t$ as the product of two factors:

$$1 + G_t = \frac{Y_{t+1}}{Y_t} = \left( \frac{A_{t+1}}{A_t} \right) \left( \frac{\kappa_{t+1}}{\kappa_t} \right)^\alpha$$

where the first factor represents the contribution of innovation and productivity growth, while the second factor represents the contribution of capital deepening. Over time, as $\kappa_t$ approaches its steady state, the second factor falls to unity, as in
the neoclassical model, so that in the steady state, capital deepening contributes nothing to growth. And as this is happening, the first factor will be increasing over time because

\[
\frac{A_{t+1}}{A_t} = 1 + g_t = 1 + (\gamma - 1)\phi(n_t)
\]

and, as we have seen, \(n_t\) increases as \(k_t\) rises to its steady-state value. Thus in the steady state, innovation and productivity growth contribute even more to economic growth, both absolutely and relative to capital accumulation, than they did in the earlier stages of growth.

### 10.4 From Manufacturing to Services

Another dramatic structural change that takes place in an advanced economy is the shift from manufacturing to services. This trend predates the outsourcing movement that many have blamed for the loss of manufacturing jobs in the United States, and it has taken place to a similar extent in all advanced economies, not just the United States. This fact suggests that there is something more fundamental at work than U.S. trade policies. This section shows how we can account for the movement using a model of nonbalanced growth developed by Acemoglu and Guerrieri (2006).

The key feature of services sectors that we focus on is that they are not as capital intensive as manufacturing sectors. To simplify the analysis, we make the extreme assumption that whereas manufacturing requires both labor and capital, services require only labor. Specifically, the output of the manufacturing sector is

\[
Y_M = AK^\alpha L_M^{1-\alpha}
\]

and the output of the service sector is\(^4\)

\[
Y_S = AL_S
\]

where \(K\) is the economy’s capital stock; and the labor inputs must add up to the population \(L\):

\[
L_M + L_S = L
\]

\(^4\) Note that we are not assuming differential technological progress in the two sectors as was done, for example, by Baumol (1967). Although it may be true that productivity growth is slower in services, it is very hard to know for certain whether it is or not given the difficulty of measuring the output of services.
We also assume that final output is produced using services and manufactures according to a production function in which they are complementary inputs. Indeed, we make the extreme assumption that services and manufactures must be combined in fixed proportions:

\[ Y = \min \{Y_M, Y_s\} \]

Then given the stock of capital \( K \), the level of productivity \( A \), and the size of the labor force \( L \), labor will be allocated across the two sectors in such a way as to result in equal output:

\[ K^\alpha L^{1-\alpha} = (L - L_M) \]

Define the economy-wide capital-labor ratio as

\[ k = K/L \]

and the manufacturing share of employment as

\[ \lambda = L_M / L \]

Then we have

\[ k^\alpha \lambda^{1-\alpha} = 1 - \lambda \]

Denote by

\[ \tilde{\lambda}(k) \]

the solution to this equation. As the economy accumulates more capital per worker, the manufacturing share of employment falls:

\[ d\tilde{\lambda}/dk < 0 \]

Indeed, as the capital-labor ratio goes to infinity, manufacturing employment falls to zero:

\[ \lim_{k \to \infty} \tilde{\lambda}(k) = 0 \]

Aggregate output can then be written as

\[ Y = AL\left[1 - \tilde{\lambda}(k)\right] \]

Suppose again that saving is a fixed proportion of output. Then,

\[ \dot{K} = sAL\left[1 - \tilde{\lambda}(k)\right] - \delta K \]

from which we have
\[ \dot{k} = sA \left[ 1 - \lambda(k) \right] - (\delta + n)k \]

Assume that the long-run growth rate \( g \) of \( A \) is positive. Then in the long run \( k \) will indeed rise without bound, manufacturing share of employment will fall to zero, and per capita income will approach

\[ Y/L = AL \left[ 1 - \lambda(\infty) \right]/L = A \]

which grows at the rate \( g \).

### 10.5 Conclusion

In this chapter we first presented the Malthusian model where agricultural output is produced with labor and a fixed factor, and where labor supply (population) in turn evolves over time at a rate that depends upon the current level of output per capita. Population increases if current per capita GDP is high, and it decreases if current per capita GDP is low. We saw that this model predicts long-term stagnation, with per capita GDP converging to the steady-state level at which population does not increase or decrease. Then we added an AK manufacturing technology to this model, and we saw that when the knowledge parameter becomes sufficiently high, people start moving from agriculture to manufacturing, at which point the economy escapes from stagnation.

In the subsequent sections, we modeled the transition from capital accumulation to innovation, and then the transition from manufacturing to services. One important limitation of these models, which makes them all appear somewhat mechanical, is that they do not capture the idea that the transition from one production mode to another may require the introduction of new institutions: new ways of regulating or deregulating markets, new financial systems, new ways of organizing firms. Private incentives to move from old to new institutions will typically not coincide with the socially optimal or growth-maximizing paths. This important issue of institutional transition will be taken up in chapter 11.

### 10.6 Literature Notes

Primary references on the transition process from Malthusian stagnation to growth are the papers by Galor and Weil (2000), Hansen and Prescott (2002), and Ashraf and Galor (2008).
While Galor and Weil emphasize demographic transition as a key feature of the industrial revolution, Mokyr and Voth (2006) stress the importance of both the development of a scientific culture among a critical mass of craftsmen and business people, and the setting up of institutions that protect property rights.

The transition from capital accumulation to innovation has been analyzed by Ha (2002). And the transition from manufacturing to services is modeled by Acemoglu and Guerrieri (2006).

For exhaustive references and comprehensive accounts of the literature and debates on the subject, we refer the reader to the *Handbook of Economic Growth* surveys by Galor (2005) and Mokyr (2005).

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**Problems**

1. Present the mechanisms that related population growth and income per capita in the Malthusian model and how they imply long-run stagnation of income per capita. Can technological progress produce sustained growth in per capita income in the setting? Explain.

2. *Skilled labor and manufacturing*

   Consider the following version of the economy studied in sections 10.2.1 and 10.2.2.

   The number of births per period is \( B = b(y)L \) where \( y \) is per capita income, \( L \) is population, \( b'(y) > 0, b(y) \to \beta \) as \( y \to \infty \), and \( b(y) \to 0 \) as \( y \to 0 \).

   Each person has a probability \( d > 0 \) of dying per period.

   Output is produced using the agricultural technology

   \[
   Y^A = A(t)L^{1-\beta}, \quad \beta \in (0, 1)
   \]

   where \( A \) is a productivity parameter with exogenous growth rate \( g \).

   a. Solve for the steady state of this economy assuming \( g > \beta(b - d) \). Is it stable? Why is the assumption necessary? How does faster productivity growth affect the steady state?

   b. Now suppose there is also a manufacturing technology:

   \[
   Y^M = A(t)S
   \]

   where \( S \) is the skilled labor supply. Each agent chooses at birth whether to become a skilled worker and work in manufacturing or an unskilled worker and work in agriculture. To become skilled an agent must pay a fixed cost \( F \). Suppose the economy faces an exogenously given constant interest rate \( r \).

   Explain why no agents born at time \( t \) will choose to become skilled if

   \[
   \int_0^t w^M(\tau)e^{-(r+\lambda)(t-\tau)}d\tau - F < \int_0^t w^A(\tau)e^{-(r+\lambda)(t-\tau)}d\tau
   \]

   where \( w^M \) is the manufacturing wage and \( w^A \) is the wage in agriculture.

   c. Assume \( g = 0 \). Find a condition on \( A \) under which all agents will choose to work in agriculture.

   d. Does there exist a level of \( A \) such that all agents will choose to become skilled?
3. **Property rights**

Suppose each agent has access to an AK production technology, \( Y_j = AK_j \), where \( Y \) is output, \( K \) is capital, \( A \) is an economy-wide productivity parameter, with growth rate \( \lambda \), and \( j \) indexes agents.

Suppose, \( \dot{K}_j = Y_j - C_j - \delta K_j - G \), where \( C \) is consumption and \( G \) is government expenditure per capita.

Assume that any agent can expropriate any other agent’s capital at cost \( s \).

a. What will the equilibrium values and growth rates of \( K_j \) and \( C_j \) be?

b. Now suppose that \( s \) depends on the level of government expenditure; for instance, it could be a measure of the extent of property rights or law enforcement. Assume \( s = \sqrt{G} \). What level of \( G \) will a benevolent government choose?

c. Assuming the existence of a benevolent government, what are the steady-state growth rates of \( K_j \) and \( C_j \)?

4. **Physical capital and innovation**

Consider an economy similar to that in section 10.3.2 except that the final good is produced using physical capital, in addition to labor and intermediate inputs, according to

\[
Y_i = \int_0^1 x_i^\alpha K_i^\beta A_i^{1-\alpha-\beta} L^{1-\alpha-\beta} di \quad 0 < \alpha, \beta < 1
\]

where \( x \) is intermediate input use, \( K \) is capital use, and \( A \) measures the quality of the intermediate input. The final good is used on a one-for-one basis for consumption, physical-capital investment, and intermediate-input production. Intermediate inputs are produced by monopolists.

To invent an intermediate of quality \( A_i = g \) with probability \( \mu \), an entrepreneur must spend

\[
N_i = \gamma A_i^{-1} \left( \frac{\mu_i^2}{2} + \delta \mu_i \right)
\]

As usual we normalize the total labor force to one.

a. Taking the amount of capital used in final-good production as given, solve for \( x_i \) and the monopolist’s profits \( \pi_i \) in equilibrium.

b. Show that the equilibrium probability of innovation in sector \( i \) is

\[
\mu_i = \lambda \left( \frac{K_i}{A_i} \right)^{\frac{\alpha}{1-\alpha}} - \delta
\]

where \( \lambda \) is a constant you should determine. How does the capital stock affect the growth rate? Will the duality of intermediate inputs tend to converge over time?

c. Suppose the economy faces an exogenously determined capital rental rate \( r \). Show that

\[
\frac{r}{\beta} = \int_0^1 \frac{2\mu_i A_i}{\alpha A_i^{1-\alpha} K_i} \frac{A_i^{1-\alpha}}{K_i} \frac{1}{\beta} di
\]

d. Solve for \( \mu_i \) in a symmetric equilibrium in which \( A_i = A_i \forall i \).

e. Suggest modeling techniques that could be used to make \( r \) endogenous.

5. **Agricultural productivity and industrialization (based on Matsuyama 1992)**

This question considers the relationship between agricultural productivity and industrialization.
Suppose there are two sectors, agriculture and manufacturing, and a single factor of production, labor. Output in manufacturing is given by

\[ Y^M(t) = A(t)[L^M(t)]^\alpha \]

while output in agriculture is given by

\[ Y^A(t) = B[L^A(t)]^\beta \]

where \( \alpha, \beta \in (0, 1) \) and manufacturing productivity \( A \) may vary over time. Suppose the total labor force is constant and normalized to one and that the labor market is competitive.

a. Solve for \( L^M(t) \) and \( L^A(t) \) as a function of \( A(t) \) and the relative price of the manufacturing good \( p(t) \).

b. Suppose intratemporal consumption preferences are defined over the consumption aggregate

\[ C(t) = [C^A(t) - \bar{C}][C^M(t)]^{1-\eta} \]

where \( \eta \in (0, 1) \) and \( \bar{C} > 0 \). Show that this implies

\[ (1-\eta)[C^A(t) - \bar{C}] = \eta p(t)C^M(t) \]

c. Use market clearing and the preceding results to show

\[ \frac{\bar{C}}{B} = [1 - L^M(t)]^\beta - \frac{\eta}{1-\eta} \frac{B}{A}[1 - L^M(t)]^{\beta-1}L^M(t) \]

d. Explain why there must be a unique solution for \( L^M(t) \).

e. How does the proportion of labor allocated to manufacturing change as agricultural productivity increases?

f. Suppose \( \dot{A}(t) = Y^M(t) \). Solve for the growth rate of \( C(t) \), and discuss how it depends on agricultural productivity. Provide some intuition for this result.

6. **Nonbalanced growth (based on Kongsamut, Rebelo, and Xie 2001)**

Consider an economy with three sectors—agriculture (\( A \)), manufacturing (\( M \)), and services (\( S \)). At time \( t \), output in the three sectors is given by

\[ A_t = B_t L^A_t X_t \]
\[ M_t = B_t L^M_t X_t \]
\[ S_t = B_t L^S_t X_t \]

where \( L^J \) denotes labor allocated to sector \( J = A, M, S \), and \( X \) is a productivity parameter with exogenous growth rate \( g \). The total labor force is constant and is normalized to one, and all goods are produced under perfect competition.

A representative consumer has preferences such that at time \( t \) she seeks to maximize

\[ U_t = (A_t - \bar{A})^\beta M_t^\gamma (S_t + \bar{S})^\theta \]

where \( \bar{A} > 0, \bar{S} > 0, \) and \( \beta + \gamma + \theta = 1 \).

a. Use labor market clearing to solve for the equilibrium prices of agriculture \( p_A \) and services \( p_S \) relative to manufacturing.
b. Use consumer optimization to show

\[
\frac{p_A(A_t - \bar{A})}{\beta} = \frac{M_t}{\gamma} = \frac{p_A(S_t + \bar{S})}{\theta}
\]

c. Define a balanced growth path as an equilibrium in which the growth rates of \(A_t, M_t,\) and \(S_t\) are constant and equal. Show that such an equilibrium does not exist.

d. Define a generalized balanced growth path as an equilibrium in which the growth rates of \(M_t\) and \(p_A A_t + p_S S_t\) are constant. Show that such an equilibrium exists if and only if \(B_S A = B_A S\).

e. Solve for \(\dot{L}^A_t, \dot{L}^M_t\) and \(\dot{L}^S_t\) in equilibrium. Discuss.
11 Institutions and Nonconvergence Traps

11.1 Introduction

*Do institutions matter for growth?* A common prediction of the growth models with endogenous innovation in the preceding chapters is that they do! For example, the analysis in chapters 3 and 4 would suggest that long-run growth would be best enhanced by some combination of good property-rights protection (to protect the rents of innovators against imitation) and a good education system (to increase the efficiency of R&D activities and the supply of skilled manufacturing labor). Our discussion of convergence clubs in chapter 7 predicts that the same policies or institutions would also increase a country’s ability to join the convergence club.

That institutions should influence economic development had already been convincingly argued by economic historians, in particular by Douglas North (see North and Thomas 1973; North 1990) and subsequently by Engerman and Sokoloff (1997, 2000). Thus, North and Thomas explain how the institutional changes brought about by trade and commercial activities led to the Glorious Revolution in 17th-century England. And North (1990) argues that the development of sedimentary agriculture followed the Neolithic revolution, which introduced communal property rights.

North (1990) defines institutions as the “rules or constraints on individual behavior” which in turn may be either formal (political constitutions, electoral rules, formal constraints on the executive, . . .) or informal (culture, social norms, . . .). Greif (1994, 2006) extends the notion of institution so that it encompasses not only the rules of the game as in North, but more generally all forms of economic organizations and finally the set of beliefs that shape the interaction between economic agents.

Two research teams over the past 10 years have made pathbreaking contributions showing the importance of institutions for economic development using historical cross-country data. A first team (see La Porta et al. 1998, 1999; Djankov et al. 2003; Glaeser et al. 2004) has emphasized legal origins as a determinant of institutions such as investors’ rights, debt collection systems, or entry regulations. A second team (see Acemoglu, Johnson, and Robinson 2001, 2002; Acemoglu and Johnson 2005) has focused on colonial origins as a determinant of a country’s institutions. These two lines of research have spurred heated debates, which we shall reflect upon in section 11.2.

*Should we recommend the same institutions to all countries?* The endogenous growth models developed in previous chapters suggest we should. In particular, they call for better property-rights protection and higher education investment.

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1. This chapter was jointly written with Erik Meyersson.