ECONOMIC BEHAVIOR UNDER UNCERTAINTY: A JOINT ANALYSIS OF RISK PREFERENCES AND TECHNOLOGY

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Abstract—A method is developed to estimate jointly risk preferences and technology under general conditions. The approach is illustrated in an application to the analysis of U.S. corn-soybean acreage decisions over time. The results provide useful information on the nature of farmers’ risk preferences and on the influence of price risk and production risk on acreage allocation and farmers’ welfare.

I. Introduction

Considerable research has focused on the influence of risk on production decisions. On one hand, the theory of the firm under uncertainty and risk aversion is now well developed, following the work of Sandmo and others. On the other hand, there is empirical evidence that most decision makers are risk averse and that price and/or revenue uncertainty has a significant influence on production decisions. Risk effects have been of special interest in agriculture where agricultural producers face considerable uncertainty due to unpredictable weather conditions and unstable agricultural markets, and where primary producers have been found to be risk averse (e.g., Dillon and Scandizzo (1978); Binswanger (1981)). These general conclusions and observations have stimulated considerable research into the effects of risk on aggregate commodity supply response, the bulk of which attempts to estimate risk effects econometrically (e.g., Behrman (1968); Just (1974); Lin (1977); Chavas and Holt (1990); Holt and Moschini (1992)). The empirical evidence to date suggests that risk has a negative and significant effect on food supply. Although this result is presumably due to farmers’ risk aversion, previous studies of aggregate supply response used a reduced form approach, an approach that does not provide precise information on the influence of risk preferences on production and producers’ welfare (Pope (1982)). As a result, the producers’ (implicit) cost of private risk bearing is still imprecisely known. In general, we might expect both the underlying technology and risk preferences to influence production decisions under risk. Consequently, there is a need to develop an approach that allows for joint estimation of the effects of technology and risk preferences on production behavior and producer welfare. Such an approach should in turn provide useful information on the nature of risk and risk preferences in resource allocation.

The objective of this paper is to present a method to estimate jointly the parameters of the risk preference function of the decision maker together with production function parameters based on observed economic behavior. We assume the decision maker behaves in a way consistent with the expected utility hypothesis. In this context, we propose a model specification and estimation scheme for production decisions under risk. Our approach is based on Full Information Maximum Likelihood (FIML) estimation of the production technology together with the first order conditions derived from expected utility maximization. The methodological approach is developed in section II. An empirical application to acreage allocation decisions for two important field crops in the U.S., corn and soybeans, is the topic of section III. Importantly, the analysis incorporates price risk, production risk, and the effects of government price support programs on the decision maker’s underlying market price distribution. Estimation and results are discussed in section IV, and section V concludes.

II. The Model

Consider a representative agent choosing a \((n \times 1)\) vector \(x = (x_1, \ldots, x_n)'\) of decision variables under uncertainty. Uncertainty is represented by a vector \(e\) of random variables with a given subjective probability distribution. Assume that the decision maker has preferences represented by the von Neumann Morgenstern utility function \(u(x, e, \alpha)\), where \(\alpha\) is a parameter vector characterizing the nature of preferences. Then, under the expected utility hypothesis, economic decisions are made as follows:

\[
\max_{x} \{Eu(x, e, \alpha): g(x, \beta) = 0, x \in R^*_n\}, \tag{1}
\]

where \(E\) is the expectation operator taken with respect to the subjective distribution of \(e\); \(g(x, \beta) = 0\) is the implicit production function which represents underlying technology; and \(\beta\) is a vector of technology parameters.

Assume that the optimization problem (1) has an interior solution denoted by \(x^* = \arg\max_{x} \{Eu(x, e, \alpha): g(x, \beta) = 0, x \in R^*_n\}\). Consider the Lagrangean \(L(x, \lambda, \alpha, \beta) = Eu(x, e, \alpha) + \lambda g(x, \beta)\), where \(\lambda\) is a Lagrangian multiplier. Under differentiability, the first-order conditions associated with the maximization problem (1) are

\[
L_x(x^*, \lambda^*, \alpha, \beta) = Eu_x(x^*, e, \alpha) + g_x(x^*, \beta)\lambda^* = 0, \tag{2a}
\]

\[
L_\lambda(x^*, \beta) = g(x^*, \beta) = 0, \tag{2b}
\]

where subscript letters denote derivatives.1

Under some regularity conditions,2 the first order conditions (2) provide a complete characterization of production behavior (e.g., Samuelson (1947)). For example, using com-

1 Throughout the paper, subscript letters denote derivatives, i.e., \(L_x = \partial L/\partial x\) = \((n \times 1)\) vector, \(L_{xx} = \partial^2 L/\partial x^2\) = \((n \times n)\) matrix, etc.

2 These regularity conditions are:
   1. the functions \(g(.)\) and \(u(.)\) are twice continuously differentiable;
   2. the matrix \(g_x = \partial g(x, \beta)/\partial x\) has rank one;
   3. the second order sufficiency condition is satisfied:

\[V L_{xx} V < 0 \quad \text{for all} \quad V \in \mathbb{R}^n \quad \text{such that} \quad V \neq 0, g_V = 0.\]

These regularity conditions are assumed satisfied throughout the paper.

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In this section we adopt the above methodology to the estimation of aggregate acreage response decisions under risk. Specifically, we focus on a time-series analysis of U.S. corn-soybean acreage allocation decisions under both price and production uncertainty, where \( n = 2, x_1 = \text{corn acreage}, \) and \( x_2 = \text{soybean acreage} \). We assume that aggregate acreage decisions are consistent with the behavior of a representative farmer given by expression parameters in \( F_1 \). Without this restriction, however, equation system (5) is not block recursive, in which case recursive estimation methods (such as those proposed by Antle (1984)) would be subject to simultaneous equation bias. In order to avoid such bias, the parameters in (5) must be estimated by a simultaneous equation method. By not imposing independence, we focus on a FIML method that allows us to investigate empirically whether or not recursive methods are appropriate in the estimation of (5).

Let \( I_t = \det \{ \partial F_t / \partial x_t \} \) be the Jacobian of the transformation given in (5), \( t = 1, \ldots, T \). Then, equation (5) is an econometric model representing a system of \( n \) implicit simultaneous equations. We propose to estimate the parameters \( \gamma = (\alpha, \beta, \sigma) \) in (5) by using Maximum Likelihood (ML) estimation techniques. The log-likelihood of the sample is

\[
L_T(x, \gamma) = \sum_{t=1}^{T} \left[ \log(f(F(x_t, \alpha, \beta, \sigma)) \right] + \log | J_t |. \tag{6}
\]

The ML estimator \( \gamma^* \) is defined to be a root of the equation \( \partial L_T(x, \gamma) / \partial \gamma = 0 \). In the present case the function \( L_T(x, \gamma) \) is nonlinear, so estimation of \( \gamma \) can only be obtained by using a numerical algorithm to maximize (6) with respect to \( \gamma \). Under fairly general conditions, the estimator \( \gamma^* \) has desirable asymptotic properties: it is a consistent and asymptotically normal estimator of the true parameters \( \gamma \) (Amemiya (1985), p. 114).

### III. An Application

In this section we adopt the above methodology to the estimation of aggregate acreage response decisions under risk. Specifically, we focus on a time-series analysis of U.S. corn-soybean acreage allocation decisions under both price and production uncertainty, where \( n = 2, x_1 = \text{corn acreage}, \) and \( x_2 = \text{soybean acreage} \). We assume that aggregate acreage decisions are consistent with the behavior of a representative farm. Also, we pay special attention to the effects of government programs on farm prices and farm income.

As a special case of equation (1), consider that the objective function of a representative farmer given by

\[
\max_a \{ Eu[\pi(x, e), a]: g(x, \beta) = 0, x \in R_+^2 \}, \tag{7a}
\]

3 Extending the analysis to allow for serial correlation appears to be a good topic for further research.
where \( \pi(x, e) \) denotes farm profit, \( x' = (x_1, x_2) \) is the vector of acreage decision for corn \((x_1)\) and soybeans \((x_2)\), and \( g(x, \beta) = 0 \) is the implicit production function with respect to acreage. Profit \( \pi(x, e) \) is defined as

\[
\pi = q'x, \quad (7b)
\]

where \( q' = (q_1, q_2) = (p_1y_1 - c_1, p_2y_2 - c_2) \) denotes the net return per acre for corn \((1)\) and soybeans \((2)\); \( p' = (p_1, p_2) \) is the vector of prices received by farmers; \( y' = (y_1, y_2) \) is the vector of yields per acre; and \( c' = (c_1, c_2) \) denotes cost of production per acre for each crop. At the time acreage decisions are made, the farmer does not know what prices \( p \) or yields \( y \) will be realized at harvest. As a result, \( p \) and \( y \) are treated as random variables that have given subjective probability distributions. Thus, the expectation operator \( E \) in \((7a)\) is over the vector of four random variables \( e = (p_1, p_2, y_1, y_2) \).

To make model \((7)\) empirically tractable, we must choose a parametric structure for \( u(\pi, \alpha) \) and \( g(x, \beta) \), and find a method to evaluate the expectation operator \( E \). Consider the \( t^{th} \) time period where profit at time \( t \), \( \pi_t \), is defined in the interval \([L, M]\). Our proposed parametric specification for \( u(\pi, \alpha) \) at time \( t \) is

\[
u(\pi_t, \alpha) = \int_L^{\pi_t} \exp(\alpha_0 + \alpha_1z + \alpha_2z^2 + \alpha_3t) \; dz, \quad (8)\]

where \( z \) is a dummy of integration, and the \( \alpha \)'s are parameters to be estimated. The time trend \( t \) in \((8)\) accounts for possible changes in risk preferences over time. The specification \((8)\) has several attractive features. First, it restricts the marginal utility of income to be positive, \( \partial u/\partial \pi = \exp(\alpha_0 + \alpha_1\pi_t + \alpha_2\pi_t^2 + \alpha_3t) > 0 \). Second, it allows for risk-averse as well as risk-loving behavior depending upon whether the utility function is concave \((\partial^2u/\partial \pi^2 < 0)\) or convex \((\partial^2u/\partial \pi^2 \geq 0)\) in \( \pi \). Third, it includes constant absolute risk aversion (CARA) as a special case when \( \alpha_2 = 0 \) (Pratt (1964); Arrow (1971)). Indeed, the Arrow-Pratt absolute risk aversion coefficient is \( AR = -\partial^2u/\partial \pi^2/\partial u/\partial \pi \) \(- (\alpha_1 + 2\alpha_2\pi_t)\), implying that \( AR \) is a constant if and only if \( \alpha_0 = 0 \). This will provide a basis for testing empirically the CARA assumption. Fourth, it provides a fairly flexible representation of risk preferences. For example, given \( \partial AR/\partial \pi = -2\alpha_2 \), it can allow for decreasing absolute risk aversion (DARA: \( \partial AR/\partial \pi < 0 \)) (see Pratt) or increasing absolute risk aversion (IARA: \( \partial AR/\partial \pi > 0 \)) depending on the sign of \( \alpha_2 \). This will provide a convenient basis to investigate empirically the DARA hypothesis suggested by Arrow.

Our proposed parametric specification for the production function \( g(x, \beta) = 0 \) at time \( t \) is

\[
g(x_t, \beta) = -x_{2t} + (\beta_0 + \beta_1[x_{1t} + DA_t] + \beta_2t + \beta_3[x_{1t} + DA_t]^2), \quad (9)\]

where \( t \) denotes the \( t^{th} \) time period, \( x_{it} \) is the acreage planted in corn \((i = 1)\) or soybeans \((i = 2)\), the \( \beta \)'s are parameters to be estimated, and \( DA_t \) is the acreage of corn diverted under government farm programs. Equation \((9)\) is quadratic in \( x_1 \); it is associated with a (strictly) concave production function if \( \beta_3(\pi) < 0 \). The time trend \( t \) in \((9)\) reflects possible structural changes in the production technology over time.

Finally, the expectation operator \( E \) in \((4)\) needs to be evaluated with respect to \( e = (p_1, p_2, y_1, y_2) \). We note, however, that government price support programs influence the subjective probability distribution of output prices \( p \) received by farmers. More specifically, a price support program places a floor under the market price. The resulting truncation affects the subjective probability distribution of prices \( p \). To see this, let \( P_i \) denote the market price of the \( i^{th} \) commodity in the absence of a price support program, and denote by \( H_i \) the corresponding price support level. Then, the price received by farmers is

\[
p_i = \begin{cases} H_i & \text{if } P_i < H_i, \\ P_i & \text{if } P_i \geq H_i, \end{cases} \quad (10)\]

where \( i = 1, 2 \). Expression \((10)\) indicates that \( p_i \) is a truncated random variable derived from \( P_i \). It has the same density function as \( P_i \), for \( P_i \geq H_i \), but the probability mass \((P_i < H_i)\) has been transferred to the truncation point \( H_i \). Here, we propose to evaluate the expectation \( E \) in \((4)\) as follows. First, the empirical joint distribution of \((P_1, P_2, y_1, y_2)\) is estimated from time series data. Second, the (truncated) distribution of \( e = (p_1, p_2, y_1, y_2) \) is derived from \((10)\) and from the distribution of \((P_1, P_2, y_1, y_2)\). Third, the expectation operator \( E \) in \((4)\) is evaluated based on the (truncated) distribution of \( e \). Because the analytical evaluation of such a truncated expectation can be quite difficult (due to the absence of a closed form solution), we propose to use numerical Monte Carlo integration to calculate \( F_1 \) in \((4)\).

Given equations \((8)\) and \((9)\) and a numerical evaluation of \( E \) in \((4)\), the method presented in section II can be implemented empirically. More specifically, the parametric specification \((8)\) and \((9)\) generates the system of simultaneous equations \((5)\) that can be estimated by using FIML techniques. This in turn is accomplished by assuming the error
term \( \nu_i \) in (5) has a joint (i.e., bivariate) normal distribution with mean zero and a finite variance-covariance matrix.\(^8\)

Before proceeding, note that additional a priori restrictions need to be imposed to render the system of equations implied by (8) and (9) estimable. The reason is that, given (8) and the fact that \( F_1 \) in (4) is in implicit form, it is possible to rescale both parameter \( \sigma_0 \) and the standard deviation of \( \nu_i \) by some positive constant without affecting this equation. In other words, the parameters of equation \( F_1 \) are not identified. To make the system of equations estimable, we impose the identifying restrictions that the variance of \( \nu_i \) equal unity. Thus, the variance-covariance matrix of \( \nu_i \) is specified as follows

\[
\text{Var}(\nu_i) = \Omega \Omega',
\]

where

\[
\Omega = \begin{bmatrix} 1 & 0 \\ \sigma_2 & \sigma_1 \end{bmatrix},
\]

\( \sigma_1 \) and \( \sigma_2 \) being parameters to be estimated. Formulation (11) expresses the variance-covariance matrix of \( \nu_i \) as the Cholesky product of the matrix \( \Omega \). This approach has the advantage of guaranteeing that \( \text{Var}(\nu_i) \) is always a positive semi-definite matrix. Note that implicit in the specification of the matrix \( \Omega \) is the identifying restriction \( \text{Var}(\nu_i) = 1 \). Also, the covariance between \( \nu_{i1} \) and \( \nu_{i2} \) is given by \( \mathbb{E}(\nu_{i1}, \nu_{i2}) = \sigma_2 \). Noting that \( \sigma_2 = 0 \) is the condition for the system of equations (5) to be recursive, we now have a convenient basis for testing the hypothesis that the parameters embedded in \( F_1 \) and \( F_2 \) can be estimated independently.

Given the parametric specification (9)–(11) and the normality of \( \nu_i \), the system of equations (5) can then be estimated by using a nonlinear algorithm to maximize log-likelihood function (6), with the Monte Carlo integration scheme used to evaluate \( \mathbb{E} \) being nested within the ML estimation algorithm.

**IV. Estimation and Results**

This section reports the application of the model to corn-soybean acreage decisions in the United States. The analysis covers the period 1954–1985 and is based on an updating of the data set reported previously in Gallagher (1978). Variables \( x_1 \) and \( x_2 \) denote acreage planted to corn and soybeans, respectively; prices are U.S. average prices received by farmers; and costs of production per acre \( (c_1 \) and \( c_2 \) are those reported by Gallagher and, for later years, in USDA sources. Following Houck et al. and Gallagher, effective support prices \( (H) \) are used to quantify the effects of government price support programs. Also, effective diversion pay-

\(^8\) Assuming that error terms are joint normally distributed is standard practice in applied econometric analyses; however, such an assumption could clearly be restrictive. Investigating alternative specifications for the joint distribution of \( \nu_i \) thus appears to be a good topic for future research.

\(^9\) The effective support prices for corn and soybeans, and the effective diversion payments for corn were constructed by Houck et al. (1976) and Gallagher (1978).

\(^10\) Note that our expectation formulations are all adaptive in nature. Although such formulations have been commonly used in previous research, they are in general not consistent with the rational expectation hypothesis. For example, the influence of government programs on the dynamics of price expectations is likely more complex than our adaptive formulations. Also, a more thorough investigation of the nature of expectations and their rationality would require modeling the “demand side” as well. Investigating such issues is left for further research.

\(^11\) The correlation coefficient between any two random variables was assumed constant throughout the sample period.
numerical methods to evaluate the expectation operator embedded in $F_1$. More specifically, $F_1$ was calculated by using Monte Carlo integration techniques, where 2000 points$^{12}$ were generated assuming that the vector $(P_1, P_2, y_1, y_2)$ is drawn from a quadrivariate normal distribution. The Monte Carlo integration routine was nested within a FIML algorithm in the maximization of equation (6).$^{13}$

The resulting maximum likelihood parameter estimates are presented in table 1. Most of the estimated parameters are significant at the 5% level. The parameter $\alpha_2$ is found to be positive and significantly different from zero. This implies that the null hypothesis of CARA preferences is rejected by the sample evidence. With $\alpha_2$, empirically appears to be a good topic for further research.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Asymptotic Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>4.0270</td>
<td>0.2545</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-16.7331</td>
<td>2.4755</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>4.5066</td>
<td>0.9574</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.2035</td>
<td>0.0380</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-2.7456</td>
<td>0.4870</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>6.6745</td>
<td>1.0867</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.0185</td>
<td>0.0010</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-3.7638</td>
<td>0.6112</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0417</td>
<td>0.0066</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>-0.0189</td>
<td>0.0120</td>
</tr>
</tbody>
</table>

Note: log likelihood = 217.10.

To interpret these coefficients, consider the Arrow-Pratt risk aversion and relative risk aversion. Since we do not differentiate between profit and terminal wealth, this distinction vanishes in our analysis. Exploring this issue empirically appears to be a good topic for further research.

To interpret these coefficients, consider the Arrow-Pratt risk aversion

1. the Arrow-Pratt absolute risk aversion coefficient $AR = - (\partial^3 u / \partial \pi^3)/(\partial u / \partial \pi)$;
2. the relative risk aversion coefficient $RR = \pi AR = - \pi (\partial^3 u / \partial \pi^3)/(\partial u / \partial \pi)$;
3. and the downside risk aversion coefficient $DR = (\partial^3 u / \partial \pi^3)/(\partial u / \partial \pi)$.

To interpret these coefficients, consider the Arrow-Pratt risk

$R = AR M_2/2 - DR M_3/6$  \hspace{1cm} (12)

where $M_i = E[(\pi - E(\pi))^i]$ is the $i^{th}$ central moment of $\pi$ (see Pratt (1964); Menezes et al. (1980)), $M_2 > 0$ being the variance of $\pi$, and $M_3$ measuring the skewness of the distribution of $\pi$. From (12), the $AR$ coefficient is a local measure of risk aversion: aversion to risk implies $AR > 0$; and larger values of $AR$ imply stronger aversion to risk (see Arrow (1971); Pratt (1964)). In addition, Pratt has shown that $\partial R / \partial E(\pi) = \text{sign}(\partial AR / \partial \pi)$ for all $\pi$, implying that the properties of $AR$ provide useful information on the effect of expected profit on the willingness to pay for risk $R$. Similar comments apply to the coefficient $RR = \pi AR$: it is a local measure of risk aversion reflecting the effect of a proportional risk on relative risk aversion as measured by $R/\pi$ (see Pratt (1964); Menezes and Hanson (1970)). Note that the $RR$ coefficient can also be written as the elasticity $RR = - \delta \ln(\delta u / \partial \pi)/\partial \pi$ which has the advantage of being “unit-free.” Finally from (12), the $DR$ coefficient is a local measure of preference toward skewness of the distribution of $\pi$. For example, “skewness to the left” means higher downside risk and a negative value for $M_3$. Thus expression (12) indicates that aversion to downside risk implies $\partial^3 u / \partial \pi^3 > 0$ and $DR > 0$ (see Menezes et al. (1980)).

Evaluated at the 1967 expected profit, the estimates of $AR$, $RR$ and $DR$ are presented in table 2 along with their standard errors.$^{15}$ The $AR$ and $RR$ coefficients are positive and significantly different from zero. This provides statistical evidence that farmers are risk averse. The large estimate of $RR$ (6.07) suggests a fairly strong degree of risk aversion.$^{16}$ The $DR$ coefficient is positive and significantly different from zero. This suggests a strong degree of risk aversion by U.S. corn-soybean farmers.

$^{12}$This choice of 2000 was one of the largest number of points feasible given the available computer resources (a 486 computer with 8 Megabytes of RAM). The results reported below were found to be fairly insensitive to the number of points used in the Monte Carlo integration.

$^{13}$In order to guarantee meaningful convergence results, the same pseudo-random variables were used as part of the Monte Carlo integration at each iteration in the maximization of the likelihood function (6). The maximum likelihood algorithm used was MAXLIK in the GAUSS programming language.

$^{14}$Menezes and Hanson (1970) distinguish between relative risk aversion and partial relative risk aversion. Since we do not differentiate between profit and terminal wealth, this distinction vanishes in our analysis. Exploring this issue empirically appears to be a good topic for further research.

$^{15}$The standard errors are calculated using the delta method.

$^{16}$Previous estimates of the relative risk aversion coefficient in agriculture have varied from 0 to over 7.5, with a median estimate around 1 (see Arrow (1971); Binswanger (1981)). Thus, our estimate of $RR$ appears to be high. This result suggests a strong degree of risk aversion by U.S. corn-soybean farmers.
different from zero. This indicates that farmers are averse to downside risk.\(^\text{17}\)

The evolution of risk preferences over the sample period is presented in Table 3 for selected years. It indicates that the absolute risk aversion (AR) coefficient has decreased between 1960 and 1975, and then increased afterwards. Table 3 also shows that the relative risk aversion (RR) coefficient has in general increased over time. Finally, it indicates some fairly large changes in the downside risk aversion (DR) coefficient over time: first a sharp decline up to the mid-1970s followed by a large increase in the 1980s.

Our results can provide useful insights on production behavior. Solving numerically the estimated equations (4) for the decision variables generates the optimal acreage allocation. This was done numerically using a Gauss-Seidel algorithm. The optimal acreage decisions for corn and soybeans were then calculated under alternative conditions in order to evaluate the implications of the model for economic adjustments. More specifically, using 1967 as the base year, we solved the first order conditions by changing selected parameters by 1%. The resulting optimal acreage was then used to calculate various elasticities of acreage response. These elasticities are reported in Table 4. In general, the estimates appear reasonable. For example, the acreage elasticities with respect to expected prices compare favorably with those reported elsewhere (e.g., Gallagher (1978); Lee and Helmberger (1985); Tegene et al. (1988)). Although always inelastic, the own price elasticities are positive while the cross price elasticities are negative. The elasticities with respect to variances are in general small. These results are generally consistent with those obtained previously by Chavas and Holt (1990). The elasticities reported in Table 4 provide more detail on acreage response than found in any previous research. For example, measuring the separate effects of yield variance and price variance on acreage decisions appears useful. Increasing yield risk is found to have a negative own effect and a positive cross effect on acreage. Also, the influence of correlations between prices and/or yields on acreage decisions is reported in Table 4.

Finally, in the absence of risk markets, our model can provide a basis for evaluating the welfare implications of risk for producers. Following Pratt, the implicit cost of risk at time \( t \) can be measured by the risk premium \( R_t \), defined implicitly as

\[
E_t[u(x_t, e_t)] = u[E_t(x_t, e_t) - R_t, \alpha],
\]

where \( E_t \) denotes the expectation operator based on the information available at time \( t \). Given the specification of the utility function given in equation (8) and the estimate of \( \alpha \) reported in Table 1, equation (13) can be solved for \( R_t \). This solution was obtained numerically using Gaussian quadrature to calculate the integral in (8) and a gradient method to solve equation (13) for \( R_t \). The resulting estimates of the risk premium are reported in Table 5 for selected years. They show that the cost of private risk bearing in corn-soybean production can be large: the risk premium \( R_t \) varies during the sample period between $200 million and $2 billion. These results indicate that risk and risk aversion can have a large negative effect on the welfare of agricultural producers.

### Table 3.—Evolution of Risk Attitudes over Time

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<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Profit ( E(\pi) )</td>
<td>0.0888</td>
<td>0.3939</td>
<td>0.4642</td>
<td>1.4463</td>
<td>0.8472</td>
<td>0.6108</td>
</tr>
<tr>
<td>Absolute Risk Aversion Coefficient ( AR = -\frac{\partial^2 u(\pi)}{\partial \pi^2} )</td>
<td>15.922</td>
<td>13.136</td>
<td>12.494</td>
<td>3.523</td>
<td>8.995</td>
<td>11.154</td>
</tr>
<tr>
<td>Relative Risk Aversion Coefficient ( RR = -\frac{\partial \pi^2}{\partial \pi} )</td>
<td>1.414</td>
<td>5.174</td>
<td>5.800</td>
<td>5.095</td>
<td>7.621</td>
<td>6.813</td>
</tr>
<tr>
<td>Downside Risk Aversion Coefficient ( DR = \frac{\partial \pi^2}{\partial \pi} )</td>
<td>262.649</td>
<td>181.679</td>
<td>165.228</td>
<td>21.547</td>
<td>90.041</td>
<td>133.550</td>
</tr>
</tbody>
</table>

Note: The estimates are evaluated at the expected profit of the corresponding year.

### Table 4.—Elasticity Estimates

<table>
<thead>
<tr>
<th>Elasticity of with respect to</th>
<th>Corn Acreage ( x_1 )</th>
<th>Soybean Acreage ( x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected corn price ( E(p_1) )</td>
<td>0.2490</td>
<td>-0.2183</td>
</tr>
<tr>
<td>Expected soybean price ( E(p_2) )</td>
<td>-0.1210</td>
<td>0.1035</td>
</tr>
<tr>
<td>Expected corn yield ( E(y_1) )</td>
<td>0.2925</td>
<td>-0.2572</td>
</tr>
<tr>
<td>Expected soybean yield ( E(y_2) )</td>
<td>-0.1210</td>
<td>0.1035</td>
</tr>
<tr>
<td>Corn price variance ( \text{Var}(p_1) )</td>
<td>-0.0335</td>
<td>0.0288</td>
</tr>
<tr>
<td>Soybean price variance ( \text{Var}(p_2) )</td>
<td>-0.00003</td>
<td>0.00003</td>
</tr>
<tr>
<td>Corn yield variance ( \text{Var}(y_1) )</td>
<td>-0.0879</td>
<td>0.0753</td>
</tr>
<tr>
<td>Soybean yield variance ( \text{Var}(y_2) )</td>
<td>0.0024</td>
<td>-0.0021</td>
</tr>
<tr>
<td>Correlation corn price ( p_1 ), soybean price ( p_2 )</td>
<td>-0.0037</td>
<td>0.0032</td>
</tr>
<tr>
<td>Correlation corn price ( p_1 ), corn yield ( y_1 )</td>
<td>-0.0086</td>
<td>0.0074</td>
</tr>
<tr>
<td>Correlation corn price ( p_1 ), soybean yield ( y_2 )</td>
<td>-0.00001</td>
<td>0.00001</td>
</tr>
<tr>
<td>Correlation soybean price ( p_2 ), corn yield ( y_1 )</td>
<td>-0.0011</td>
<td>0.0009</td>
</tr>
<tr>
<td>Correlation soybean price ( p_2 ), soybean yield ( y_2 )</td>
<td>-0.0012</td>
<td>0.0011</td>
</tr>
<tr>
<td>Correlation corn yield ( y_1 ), soybean yield ( y_2 )</td>
<td>-0.0013</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Note: The elasticities are evaluated at the 1967 data point.

### Table 5.—Evolution of Risk Premium \( R_t \) over Time

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Risk Premium ( R_t )</td>
<td>0.782</td>
<td>2.081</td>
<td>0.593</td>
<td>0.219</td>
<td>0.238</td>
<td>0.814</td>
</tr>
</tbody>
</table>

(billion 1967 dollars)
ECONOMIC BEHAVIOR UNDER UNCERTAINTY 335

relies on FIML estimation of all the parameters of the production function and of the first order conditions associated with the maximization of expected utility. The method is applied to U.S. aggregate corn and soybean acreage response decisions, incorporating both price risk and production risk. Particular attention is given to the effects of government price support programs that truncate the price distribution and thus reduce price risk. The empirical analysis relies heavily on numerical methods: for evaluating the expectations in the first order conditions; for maximizing the log likelihood function; for solving the first order conditions for optimal acreage decisions; and for estimating the implicit cost of risk. The obvious advantage of our approach is that the methods we employ do not require the existence of closed form solutions, and thus can handle different assumptions about the form of the utility function or the nature of the uncertainty facing the decision maker. For example, our analysis handles a quadrivariate truncated risk distribution (involving two truncated prices and two yields). This indicates that our proposed method is flexible and appears promising in the joint analysis of technology and risk preferences.

The empirical results for corn and soybean acreage indicate that corn-soybean farmers are risk averse and that they exhibit decreasing absolute risk aversion and downside risk aversion. The analysis also provides useful information on the influence of risk on acreage decisions and on the farmers’ implicit cost of private risk bearing. This suggests that the proposed method can help refine the economic analysis of production behavior under uncertainty. It is hoped that it will help stimulate further research on government farm programs and their influence on the efficiency of risk allocation in agriculture.

REFERENCES


