The Peanut Butter Jelly Time (PBJT) company is a firm that produces peanut butter (Interestingly, they don’t produce any jelly!). After hiring a consulting company to analyze their production, PBJT finds that their total and marginal costs of production are given by the following equations relating costs to the level of output:

\[ TC = 3Q - Q^2 + (1/6)Q^3 \]
\[ MC = 3 - 2Q + (1/2)Q^2 \]

1. Does the company have any fixed costs? How can you tell? Given this level of fixed costs, will average fixed costs change as input increases? If so, how do they change?

No, the firm has no fixed cost (all components of the TC have Q in them) \(\rightarrow\) no average fixed cost either.

2. Find the level of peanut butter output where ATC is equal to MC (Hint: in mathematics, a quadratic equation has the general form: \(aZ^2 + bZ + c = 0\) where the constants a, b and c are respectively known as the quadratic term, linear term and constant. There are two values that result in the above equality (i.e. \(Z^*\)). If you want to find these two values, the following result is used:

\[ Z^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

where the values of a, b, and c are obtained from the quadratic equation shown above.) Draw the general shapes of the ATC and MC curves in a graph (don’t worry about exact numbers). Where is the value you just found on this graph (where ATC=MC).

We have:

\[
\frac{ATC}{Q} = \frac{TC}{Q} = \frac{3Q - Q^2 + \frac{Q^3}{6}}{Q} = 3 - Q + \frac{Q^2}{6}
\]

Set ATC = MC, we have:

\[
3 - Q + \frac{Q^2}{6} = 3 - 2Q + \frac{Q^2}{2}
\]

\[
Q - \frac{2Q}{6} = 0
\]

To find Q, you can either use the formula given, or do as follow (recognizing Q appears in both elements of the formula):

\[
Q \left(1 - \frac{Q}{3}\right) = 0
\]

\(\Rightarrow\) Q = 0 or Q = 3

\(\Rightarrow\) Plug those values back to ATC and MC, to find their corresponding values.

At \(Q = 0\):

\[
ATC_{Q=0} = MC_{Q=0} = 3
\]
At $Q = 3$:

$ATC_{Q=3} = MC_{Q=3} = 3/2$

$\Rightarrow$ MC and ATC cross at two points. We know one of the points is where ATC is at minimum.

At $Q = 3$, $ATC_{\min} = 3/2$ (Why not at $Q = 0$?)

The curves should look like below.

3. This firm’s costs have the interesting property that ATC=AVC. Why is this?
   The firm has no fixed cost $\Rightarrow$ AFC = 0 or ATC = AVC

4. Professor Gould has mentioned that it is possible for production to be at the profit maximizing level, and yet profits may still be negative. Is this a possibility for this firm? What does this imply about the shutdown level of production and the breakeven level of production.

The reason why some firm still produces even though it is making a loss is by doing that, it minimizes its loss by covering some part of total fixed cost. This is not the case here because this firm has no fixed cost at all.

ATC = AVC also implies $Q_{BE} = Q_{SD} \Rightarrow$ Either the firm shuts down, or makes a non-negative profit.

5. Suppose the market price of a jar of peanut butter is $4. How much profit is the firm making at this price?

*Profit-maximizing Output level: $P = MR = MC \Rightarrow$

$$3 - 2Q + \frac{Q^2}{2} = 4$$
Using the formula given, you get:

\[ 3 - 2Q + \frac{Q^2}{2} = 4 \]

The profit-maximizing output level is

\[ Q = 2 \pm \sqrt{6} \]

(You should be able to explain why we won’t choose the other value).

**Maximum Profit:** \( \Pi = TR - TC \)

\[ TR = PQ^* = 4(2 + \sqrt{6}) \]

\[ TC = 3Q^* - Q^{*2} + \frac{Q^{*3}}{6} = 3(2 + \sqrt{6}) - (2 + \sqrt{6})^2 + \frac{(2+\sqrt{6})^3}{6} = \frac{10+6\sqrt{6}}{3} \]

\[ \Pi = \frac{14 + 6\sqrt{6}}{3} \]